18.06 Problem Set 1

due Thursday, September 11, 2014, before 4:00 pm (sharp deadline) in Room E17-131

Write down all details of your solutions, not just the answers. Show your reasoning. Please staple the pages together and **clearly write your name**, your recitation section, and the name of your recitation instructor on the first page of the problem set.

Cooperation on problems is permitted, but all solutions must be written up independently and you must list your collaborators on the problem set. You should first try to solve each problem yourself, otherwise you will not learn much from hearing the solution.

Please note that the problems listed below are out of the 4th edition of the textbook. Please make sure to check that you are doing the correct problems.

Problem 1. Section 1.2, Problem 13, page 20.

Problem 2. Section 2.1, Problem 7, page 41.

Problem 3. Section 2.2, Problem 12, page 53.

Problem 4. Section 2.3, Problem 19, page 65.

Problem 5. Section 2.4, Problem 6, page 76.

Problem 6. Section 2.5, Problem 11, page 90.

Problem 7. (a) Calculate $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, for $n = 1, 2, 3, 4$.

(b) The sequence of *Fibonacci numbers* $(f_1, f_2, f_3, \dots) = (1, 1, 2, 3, 5, 8, 13, \dots)$ is the sequence of integers such that $f_1 = f_2 = 1$ and

$$f_n = f_{n-1} + f_{n-2}, \qquad \text{for } n \geq 3.$$

How is your answer to part (a) related to the Fibonacci numbers? Predict the answer to part (a) for $n = 5$ and then check it by calculation.

Problem 8. (a) Can a matrix of the form $\begin{pmatrix} 0 & a & 0 \\ b & c & d \\ 0 & e & 0 \end{pmatrix}$ have an inverse? (Explain why not or present an example.)

(b) What about a matrix of the form $\begin{pmatrix} a & 0 & b \\ 0 & 0 & c \\ d & e & 0 \end{pmatrix}$?

Problem 9. For a linear system with 2 variables in 2 indeterminates, a temporary failure in Gaussian elimination can only happen at the first step – namely, when the leftmost coefficient of the first equation is 0. See Example 3 on page 47 for an example.

For a linear system with 3 variables in 3 indeterminates, which of the following can happen?

(a) a temporary failure at the first step (i.e., the first pivot position is zero);

(b) a temporary failure at the second step (i.e., the second pivot position is zero);

(c) a temporary failure at the third step (i.e., the third pivot position is zero)?

Note that we are speaking of **temporary** failures only, so the system should still have a unique solution (that can be found by permuting the rows when the pivoting fails).

When your answer is "No", give a reason; when it is "Yes", show an example.

Problem 10. (Computational Problem) For this problem, you can use `Julia`, `Matlab`, `Mathematica`, `Maple`, `Python`, `Sage`, or any other computer language that you like. Please check `http://web.mit.edu/18.06/www/Fall14/ps1c.html` for some info on `Julia`.

The problem itself is available at
`http://web.mit.edu/18.06/www/Fall14/ps1c.pdf`

Attach a printout to your solutions of the problem set.