18.06 **EXAM 3** December 3, 2014

Your PRINTED name is: ____________________

Please circle your recitation:

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R02	T 10	E17-136	Darij Grinberg
R03	T 10	24-307	Carlos Sauer
R04	T 11	24-307	Carlos Sauer
R05	T 12	E17-136	Tanya Khovanova
R06	T 1	E17-139	Michael Andrews
R07	T 2	E17-139	Tanya Khovanova

Grading	
1	
2	
3	
4	
Total:	

Each problem is 25 points, and each of its five parts (a)–(e) is 5 points.

In all problems, write all details of your solutions. Just giving an answer is not enough to get a full credit. Explain how you obtained the answer.

Problem 1. Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(a) Find the eigenvalues λ_1 and λ_2 of A.

(b) Solve the initial value problem $d\mathbf{u}/dt = A\,\mathbf{u}\,,\ \ \mathbf{u}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(c) Find a diagonal matrix which is similar to the matrix A.

(d) Find the singular values σ_1 and σ_2 of A.

(e) Is the matrix A positive definite?

Problem 2. True or false? If your answer is "true", explain why. If your answer is "false", give a counterexample.

(a) Every positive definite matrix is nonsingular.

(b) If A is an $n \times n$ matrix with real eigenvalues and with n linearly independent eigenvectors which are orthogonal to each other, then A is symmetric.

(c) If a matrix B is similar to A, then B has the same *eigenvectors* as A.

(d) Any symmetric matrix is similar to a diagonal matrix.

(e) Any matrix which is similar to a diagonal matrix is symmetric.

Problem 3. (a–c) Consider the matrix $A = \begin{pmatrix} 2 & t & 0 \\ t & 2 & t \\ 0 & t & 2 \end{pmatrix}$ that depends on a parameter t.

(a) Find all values of t, for which the matrix A has 3 nonzero eigenvalues.

(b) Find all values of t, for which the matrix A has 3 positive eigenvalues.

(c) Find all values of t, for which the matrix A has 3 negative eigenvalues.

(d) Find a singular value decomposition $B = U\Sigma V^T$ for the matrix $B = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$.

(e) Find orthonormal bases of the 4 fundamental subspaces of the matrix B from part (d).

Problem 4. (a–d) Consider the following operations on the space of quadratic polynomials $f(x) = ax^2 + bx + c$. Which of them are linear transformations?

If they are linear transformations, find their matrices in the basis $1, x, x^2$.

If they are not linear transformations, explain it using the definition of linear transformation.

(a) $T_1(f) = f(x) - f(1)$.

(b) $T_2(f) = f(x) - 1$.

(c) $T_3(f) = x - f(1)$.

(d) $T_4(f) = x^2 f(1/x)$.

(e) The linear transformation $R : \mathbb{R}^2 \to \mathbb{R}^2$ is the reflection with respect to the line $x+y = 0$. Find the matrix of R in the standard basis of $\mathbb{R}^2$.

If needed, you can use this extra sheet for your calculations.

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