18.06 **EXAM 2** November 5, 2014

Your PRINTED name is: ____________________

Please circle your recitation:

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R06	T 1	E17-139	Michael Andrews
R07	T 2	E17-139	Tanya Khovanova

Grading

1	
2	
3	
4	
Total:	

Each problem is 25 points, and each of its five parts (a)–(e) is 5 points.

In all problems, write all details of your solutions. Just giving an answer is not enough to get a full credit. Explain how you obtained the answer.

Problem 1. (a) Do Gram-Schmidt orthogonalization for the vectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

(Find an orthogonal basis. Normalization is not required.)

(b) Find the $A = QR$ decomposition for the matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$.

(c) Find the projection of the vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ onto the line spanned by the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(d) Find the projection of the vector $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ onto the plane $x + y + z = 0$ in $\mathbb{R}^3$.

(e) Find the least squares solution $\widehat{\mathbf{x}}$ for the system $\begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 10 \\ 0 \end{pmatrix}$.

Problem 2. Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$. (a) Calculate the determinant $\det(A)$.

(b) Explain why A is an invertible matrix. Find the entry $(2,3)$ of the inverse matrix A^{-1}.

(c) Notice that all sums of entries in rows of A are the same. Explain why this implies that $(1,1,1)^T$ is an eigenvector of A. What is the corresponding eigenvalue λ_1?

(d) Find two other eigenvalues λ_2 and λ_3 of A.

(e) Find the projection matrix P for the projection onto the column space of A.

Problem 3.

(a) Calculate the area of the triangle on the plane $\mathbb{R}^2$ with the vertices $(1,0)$, $(0,1)$, $(3,3)$ using determinants.

(b) Find all values of x for which the matrix $A = \begin{pmatrix} 1 & x \\ 1 & 1 \end{pmatrix}$ has an eigenvalue equal to 2.

(c) Diagonalize the matrix $B = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$.

(d) Calculate the power B^{2014} of the matrix $B = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$.

(e) Let Q be any matrix which is symmetric and orthogonal. Find Q^{2014}. Explain your answer.

Problem 4. Consider the Markov matrix $A = \begin{pmatrix} 0 & 1/3 & 1/3 & 0 \\ 1/2 & 0 & 1/3 & 1/2 \\ 1/2 & 1/3 & 0 & 1/2 \\ 0 & 1/3 & 1/3 & 0 \end{pmatrix}$.

(a) Three of the eigenvalues of A are $1, 0, -1/3$. Find the fourth eigenvalue of A.

(b) Find the determinant $\det(A)$.

(c) Find the eigenvector of the transposed matrix A^T with the eigenvalue $\lambda_1 = 1$.

(d) Find the eigenvector of the matrix A with the eigenvalue $\lambda_1 = 1$. (Hint: Notice that nonzero entries in each column of A are the same.)

(e) Find the limit of $A^k \, (1\ 0\ 0\ 0)^T$ as $k \to +\infty$.

If needed, you can use this extra sheet for your calculations.

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