18.06 **EXAM 1** October 1, 2014

Your PRINTED name is: ________________________

Please circle your recitation:

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R07	T 2	E17-139	Tanya Khovanova

Grading

1	
2	
3	
4	
Total:	

Each problem is 25 points, and each of its five parts (a)–(e) is 5 points.

In all problems, write all details of your solutions. Just giving an answer is not enough to get a full credit. Explain how you obtained the answer.

Problem 1. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \\ 3 & 7 & 10 \end{pmatrix}$. (a) Find the $A = LU$ factorization of the matrix A.

(b) Solve the system $A\mathbf{x} = (3, 10, 20)^T$.

Let $B = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \\ 3 & 7 & k \end{pmatrix}$ (obtained by replacing the bottom right entry of A by parameter k).

(c) For which values of k is the matrix B singular?

(d) Find all values of k for which the system $B\mathbf{x} = (1, 2, 3)^T$ has infinitely many solutions. (You don't need to solve the system in this part.)

(e) Find all values of k for which the system $B\mathbf{x} = (10, 1, 2014)^T$ has exactly one solution. (You don't need to solve the system in this part.)

Problem 2. Which of the following sets of vectors are vector subspaces of $\mathbb{R}^3$? Explain your answer?

(a) All vectors $(x, y, z)^T$ such that $10\,x + y + 2014\,z = 0$.

(b) All vectors $(x, y, z)^T$ such that $x + y + z \leq 2014$.

(c) All vectors $(x, y, z)^T$ such that $x + y + z = 0$ AND $x + 2y + 3z = 0$.

(d) All vectors $(x, y, z)^T$ such that $x + y + z = 0$ OR $x + 2y + 3z = 0$.

(e) All vectors $(b_1, b_2, b_3)^T$ such that the system $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \mathbf{x} = (b_1, b_2, b_3)^T$ has a solution.
(You don't need to solve the system in this part.)

Problem 3. Let $A = \begin{pmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 3 \\ -1 & -2 & 0 & 2 & 3 \end{pmatrix}$.

(a) Find the complete solution of $A\mathbf{x} = \mathbf{0}$.

(b) Find the complete solution of $A\mathbf{x} = (1, 2, 0)^T$.

(c) Find the linear condition(s) on b_1, b_2, b_3 that guarantee that the system $A\mathbf{x} = (b_1, b_2, b_3)^T$ has a solution.

(d) Find the rank of A and dimensions of the four fundamental subspaces of A.

(e) Find bases of the four fundamental subspaces of A.

Problem 4. Which of the following statements are true? Explain your answer.

(a) Matrices A and $R = RREF(A)$ always have the same column space $C(A) = C(R)$.

(b) Matrices A and $R = RREF(A)$ always have the same row space $C(A^T) = C(R^T)$.

(c) If A is an $m \times n$ matrix with linearly independent columns, then $m \geq n$.

(d) If A is an $m \times n$ matrix of rank $r = m$, then the left nullspace $N(A^T)$ contains only the zero vector $\mathbf{0}$.

(e) If two $m \times n$ matrices A and B have exactly the same 4 fundamental subspaces

$$C(A) = C(B), \quad N(A) = N(B), \quad C(A^T) = C(B^T), \quad N(A^T) = N(B^T),$$

then $A = B$. (Prove that $A = B$ or give a counterexample where $A \neq B$.)

If needed, you can use this extra sheet for your calculations.

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