# 18.06 (Fall '13) Problem Set 6

This problem set is due Thursday, October 24, 2013 by 4pm in E17-131.

1. Do Problem 3 from 8.5.
2. Do Problem 4 from 8.5.
3. Do Problem 7 from 5.1.
4. Do Problem 18 from 5.1.
5. Do Problem 29 from 5.1.
6. Do Problem 16 from 5.2.
7. Do Problem 12 from 5.2.
8. Do Problem 28 from 5.3.
9. This problem is about Legendre polynomials.

   One can check orthogonality of polynomials the old fashioned way by doing integration the calculus way. The point of this problem is to show that it can also be done by what is known as monte carlo integration, where we use random numbers. Create random numbers x uniformly on [-1,1] by taking x=rand(100000)*2-1 in Julia or in MATLAB rand(100000,1)*2-1. Consider the polynomials $p_0(x) = 1, p_1(x) = x, p2(x) = 3 * x^2 - 1, p3(x) = 5x^3 - 3x$. You can evaluate p3, for example, in julia

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   p3=5x.^3-3x
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   and in MATLAB with

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   5*x.^3-3*x.
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   By typing mean(pi.*pj) for all i and j between 0 through 3 verify the orthogonality. You may have to add some 0's to see convergence. By numerical experiment what is $\|p_i\|^2$ for $i = 0, 1, 2,$ and 3? (Hint, the reciprocal may be useful to look at.)(Note: the normalization here may be different from that found elsewhere,)

10. This problem will not have a definitive answer. We just ask you to try something reasonable. We do not know the answer ourselves. Let $A$ be a 5 x 5 matrix whose entries have absolute value all less than or equal to 1. See how big you can make the absolute value of the determinant. (You can try generating lots of matrices such as A=2*rand(5,5)-1 and see how high you can get, by saving the biggest one, or you can try tweaking ones that are large to make them larger, or if you know how you can try to do fancier things with optimization. You will be graded for trying something not for finding the optimum. I will ask the graders to let us know who finds the biggest one. (Note: please do not tie up ijulia.csail for long runs, better to use athena or your own computer.)