18.06 (Fall '13) Problem Set 5

This problem set is due Thursday, October 17, 2013 by 4pm in E17-131.

1. In Section 4.2 of the textbook, you learned that if p is the projection of the vector b onto the line a, then p is characterized by the fact that the line from p to b is perpendicular to p. One might guess that this criterion extends to projections onto subspaces of dimension > 1, but this is incorrect: In this question you'll demonstrate, by example, that this approach leads to infinitely many possible "projections". (The right criterion is that the line from p to b is perpendicular to every column of A.)

   a) Let A be an $m \times n$ matrix, and let b be a vector in $\mathbb{R}^m$. We'd like to find the projection of b onto the column space of A. If $p = Ax$ is in the column space of A, show that the equation x must satisfy for the line from b to p to be perpendicular to p is

   $$x^T A^T b = x^T A^T A x.$$

   b) Now suppose for example A is the $m \times 2$ matrix

   $$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ \cdots & \cdots \\ 0 & 0 \end{pmatrix}.$$

   Show that in this case, the above equation is just the equation of a circle. Describe clearly the circle.

   We'd like to have a unique projection, not a whole circle's worth of them. Thus we must insist that the line from b to p be perpendicular to the entire column space of A.

2. Do Problem 9 from 4.3.

3. Do Problem 10 from 4.3.

4. Do Problem 12 from 4.3.

5. Do Problem 13 from 4.3.

6. Do Problem 4 from 4.4.

7. Do Problem 18 from 4.4.

8. Do Problem 37 from 4.4. To rephrase: Q has othonormal columns. We want to perform Gram-Schmidt on

   $$[Qa]$$

   and we only need to change the final column.

9. Use Julia or otherwise to compute the coefficients of a best least squares fifth degree approximation to $y = \sin(x)$ on $[0, 2\pi]$.

   In Julia you can execute the following code.

```
t=2*pi*(0:.01:1)
A = [t[i]^k for i=1:length(t),  k=0:1:5];
c=float(A)\sin(t)
```

   If you would like to see the approximation, you can evaluate the polynomial and plot it:

```
x=(0:.001:1)*2*pi
z=0*x;
for i=length(c):-1:1
  z=z.*x+c[i];
end

using PyPlot
plot(x,z)
plot(x,sin(x))
```

10. Compare the quintic above to the best solution obtainable from a Taylor series expansion of $\sin x$: $x - x^3/6 + x^5/120$. Also compare with the Taylor series about $x = \pi$: $-(x-\pi) + (x-\pi)^3/6 - (x-\pi)^5/120$.