September 26, 2013 Alexander Dubbs

18.06 Set 3 Solutions

1.

$$A \longrightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \\ 4 & 8 & 16 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 9 & -4.5 \\ 1 & 3 & -1.5 \\ 2 & 6 & -3 \end{bmatrix}$$

$$M = \begin{bmatrix} a & b \\ c & bc/a \end{bmatrix}$$

3. Define e_i to be the i-th column of I, which is $n \times n$. The nullspace matrix of A has columns $e_i - e_{i+n}$ for $i = 1, 2, \ldots, n$. The nullspace matrix of B is the same. The nullspace matrix of C has columns $e_i - e_{i+n}$ for $i = 1, 2, \ldots, n$ in addition to columns $e_{i+n} - e_{i+2n}$ for $i = 1, \ldots, n$, a total of $2n$ columns.

4.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

5. The rref of A is

$$\begin{bmatrix} 1 & 0 & -20/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 0 \end{bmatrix},$$

therefore the rank is 2. The rref of A^t is

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix},$$

therefore the rank is 2. Partially reducing A with q we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & q-2 \end{bmatrix},$$

so if $q \neq 2$, the matrix has rank 3, and otherwise it has rank 2. Partially reducing A^t with q we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & q-2 \end{bmatrix},$$

which is the same situation.

6. a. $(1,1,1,1)^t$, b. It is $(1,-1,0,0)^t$, $(0,1,-1,0)^t$, $(0,0,1,-1)^t$. c. $(0,0,-1,1)^t$, $(1,-1,-1,0)^t$. d. The column space has basis $(1,0,0,0)^t$, $(0,1,0,0)^t$, $(0,0,1,0)^t$, $(0,0,0,1)^t$, the basis of the nullsapce is the empty set.

7. Let

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The trick is $A_1 + A_5 + A_6 = A_2 + A_3 + A_4$ which is the matrix of all 1's, so $A_1 = A_2 + A_3 + A_4 - A_5 - A_6$. Now let $c \in \mathbb{R}^{\ntrianglelefteq}$ such that $c_2A_2 + c_3A_3 + c_4A_4 + c_5A_5 + c_6A_6 = 0$ (there is no c_1, c is indexed from 2 to 6). c_2 must be 0 since A_2 has a 1 in the lower right hand corner and no other A_i has that for $i \neq 1$. Likewise c_3 must be 0 since A_3 has a 1 in the upper left hand corner and no other A_i has that for $i \neq 1$. $c_4 = 0$ since A_4 has a 1 in the dead center. It is obvious that A_5 and A_6 are linearly independent from each other, hence $c_5 = c_6 = 0$. So $c = \vec{0}$ and the matrices are linearly independent.

8. a. $y = c$, a constant. b. $y = 3x$. c. $y = 3x + c$.

9. Integer matrices: rank zero 0%,