18.06 (Fall '13) Problem Set 3

This problem set is due Thursday, September 26, 2013 by 4pm in E17-131.

1. Do Problem 3 from 3.3.
2. Do Problem 8 from 3.3.
3. Do Problem 24 from 3.3.
4. Do Problem 10 from 3.4.
5. Do Problem 18 from 3.4. For the second matrix, your answer should specify which values of q give which rank.
6. Do Problem 16 from 3.5.
7. Do Problem 41 from 3.5.
8. Do Problem 31 from 3.5.
9. Let's look at the ranks of two kinds of random matrices experimentally on the computer. In the first case, we will compute the ranks of random 3×3 matrices with *integer* entries 0 through 5.

   In julia we can get, say, 10,000 (you can do more) samples by typing

   ```
   v=[rank(rand(0:5,3,3)) for i = 1:10000];
   ```

   and the counts for rank 0, rank 1, rank 2 and rank 3 by

   ```
   [sum(v.==j) for j=0:3]
   ```

   Now do the same for non-integer random numbers by typing

   ```
   [v = [rank(randn(3,3)) for i=1:10000];
   ```

   (These are random matrices from the so-called normal distribution, whose entries will almost certainly not be integers).

   The answers are very different because there are just so many directions possible when real numbers are allowed.

10. For any n, consider the $n \times n$ matrix

$$M_n = \begin{pmatrix} 1 & 2 & \cdots & n \\ n+1 & n+2 & \cdots & 2n \\ & \cdots & & \\ n(n-1)+1 & n(n-1)+2 & \cdots & n^2 \end{pmatrix}$$

   ($n = 3$ was covered in class). Use a computer to find the rank of M_n for n between 1 and 25. In julia, you can use

```
r(n)=reshape(1:n^2,n,n)
```

to generate the matrices (or transposes) and

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[rank(r(n)) for n=1:25]'
```

to output the vector of ranks. It should be easy to see experimentally what the answer is. Perhaps try to prove it on your own or in a group, but you need not include a proof.

18.06 Wisdom. Matrices are often full rank in the real world, unless there is hidden or not so hidden structure.