September 20, 2013 Alexander Dubbs

18.06 Set 2 Solutions

1. First matrix:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

Second matrix:

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Let $x \in \mathbb{R}^n$, the answer is $(x_1, x_1, x_2, x_2, \ldots, x_n, x_n)$.

3. a. $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$. b. $\begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$. c. $\begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

4. a. True b. True c. False, consider $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$.

5. Let c be a scalar. Let $s_0, s_1 \in S$ and $t_0, t_1 \in T$. $c(s_0 + t_0) = (cs_0) + (ct_0)$. Each of cs_0 and ct_0 are in S, T, respectively since they are vector spaces, so $c(s_0 + t_0) \in S + T$. Also, $(s_0 + t_0) + (s_1 + t_1) = (s_0 + s_1) + (t_0 + t_1)$. Each of $s_0 + s_1$ and $t_0 + t_1$ are in S, T respectively, so $(s_0 + t_0) + (s_1 + t_1) \in S + T$.

6. Let $a_1, \ldots, a_m$ be the columns of A and $b_1, \ldots, b_n$ be the columns of B. The set of columns of M are $a_1, \ldots, a_m, b_1, \ldots, b_n$. Proof: First we show that any element of the column space of M is in $S + T$. Every element of the column space of M is a linear combination of a_i's and b_j's, which is a linear combination of a_i's plus a linear

combination of b_j's. The former is in S, the latter is in T, so the whole thing must be in $S+T$. Furthermore, if s_0+t_0 is in $S+T$, it must be in the column space of M. That's because s_0 is a linear combination of a_i's and t_0 is a linear combination of b_j's, so their sum is a linear combination of a_i's and b_j's, so it's in the column space of M.

7. We need to find two row vectors r_1 and r_2 which are linearly independent and both have zero dot product with both $(2,2,1,0)$ and $(3,1,0,1)$. The correct way to do this involves the Gram-Schmidt process, which you will soon learn. The Mathematica function QRDecomposition is helpful. The two vectors are $r_1=(0,1,-2,-1)$, $r_2=(-6,7,-2,11)$, and the correct matrix is r_1 stacked on top of r_2.

8. a.

$$\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

x_1 and x_3 are pivots, x_2, x_4, x_5 are free.
b.

$$\begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

x_1 and x_2 are pivots, x_3 is free.

9. They do form a vector space, since any constant times an element of the space is in the space, and the sum of any two elements in the space is in the space. (i) and (iii). If $f(x_0)=0$ for some x_0, $cf(x_0)=0$, and $f_1(x_0)+f_2(x_0)=0$. (ii) and (iv). If $f(x_0)=d\neq 0$, $2f(x_0)=2d\neq d$, so it is not a vector space.

10. a.

$$\begin{bmatrix} 0 & 4 \\ 8 & 3 \\ 6 & 1 \\ 7 & 3 \end{bmatrix}$$

b.

```
A = randn(m,n); B = randn(n,p);
for integer choices of m,n,p. Then do
(A*B)'-B'*A'
inv(A*B) - inv(B)*inv(A)
inv((A*B)') - inv(A')*inv(B')
P = [[0,0,1];[0,1,0];[1,0,0]];
P^(-1) - P'
```