18.06 (Fall '13) Problem Set 2

This problem set is due Thursday, September 19, 2013 by 4pm in E17-131. The problems are out of the 4th edition of the textbook. For computational problems, a printout may be useful if convenient.

1. Do Problem 22 from 2.7.
2. Find all $2n$-dimensional vectors

$$v = (v_1, v_2, \cdots, v_{2n})$$

   such that $P_n v = v$, where P_n is the block permutation matrix

$$\begin{pmatrix} S & & & 0 \\ & S & & \\ & & \ddots & \\ 0 & & & S \end{pmatrix}$$

   where S is the 2×2 permutation matrix

$$S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

   For example, P_1 is just S and

$$P_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3. Do Problem 17 from 2.7.
4. Do Problem 18 from 3.1.
5. Do Problem 30 part a) from 3.1. (You just have to verify parts (i) and (ii) from page 122).
6. Do Problem 31 from 3.1.
7. Do Problem 21 from 3.2.
8. Do Problem 1 from 3.2.
9. Consider all functions of the form

$$f(x) = a + b \sin x + c \cos x.$$

   Do they form a vector space? What about those functions $f(x) = a + b \sin x + c \cos x$ for which

(i) $f\left(\frac{\pi}{10}\right) = 0$?

(ii) $f\left(\frac{\pi}{10}\right) = \frac{\pi}{10}$?

(iii) $f(0) = 0$?

(iv) $f(0) = 1$?

10. (a) The following problem does not require a computer or knowledge of Julia.

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Julia has a very nice way to construct matrices:
In: [i+j for i=1:5, j=1:5]
Out:  5x5 Array{Int64,2}:
 1   2   3   4   5
 2   4   6   8  10
 3   6   9  12  15
 4   8  12  16  20
 5  10  15  20  25

Here is a random matrix A
In: A=rand(0:9,2,4)
Out: 2x4 Array{Int64,2}:
 0  8  6  3
 4  3  1  7

This reconstructs A
In: [A[i,j] for i=1:size(A,1), j=1:size(A,2)]
Out: 2x4 Array{Int64,2}:
 0  8  6  3
 4  3  1  7

What matrix does this construct?
In:[A[i,j] for  j=1:size(A,2), i=1:size(A,1)]
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(b) Demonstrate in Julia or your favorite computer language a few examples illustrating

$$(AB)^T = B^T A^T$$
$$(AB)^{-1} = B^{-1} A^{-1}$$
$$((AB)^T)^{-1} = (A^T)^{-1}(B^T)^{-1}$$
$$P^{-1} = P^T$$

where P is a permutation matrix. (In Julia or Matlab, A' denotes transpose and inv(A) denotes inverse.)

18.06 Wisdom. Knowing one way to do something is good. Knowing many ways to do something is better.