18.06 (Fall ’13) Problem Set 1

This problem set is due Thursday, September 12, 2013 by 4pm in E17-131. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set. Some of you may be succesful installing Julia from the “Julia downloads” web page. In a few days, we are hoping to set up a web page like you saw in class, so that no installation is needed. Stay tuned.

1. Do Problem 19 from 2.1.

2. Do Problem 19 from 2.2.

3. The graph of

   $$y = a + bx + c \sin\left(\frac{\pi x}{2}\right)$$

   passes through the points $(0, 2)$, $(1, 6)$ and $(2, 12)$. Set up and solve a matrix equation for the variables (a, b, c).

4. Do Problem 20 from 2.4.

5. Do Problem 23 from 2.4.

6. Do Problem 3 from 2.5.

7. Do Problem 6 from 2.6.

8. Open your preferred computer algebra software. For various n, say $n = 1, 2, 3, 4, 5$, create the upper triangular $n \times n$ matrix of ones on the diagonal and the triangle above. (In Julia and MATLAB this is triu(ones(n,n)) and triu(ones(n)) also works in MATLAB). We would like you to square each of these matrices and tell us the pattern that you see.

   In Julia, one can obtain the results for $n = 1, 2, 3, 4, 5$ by typing for example

   ```
   {triu(ones(n,n))^2 for n=1:5}
   ```

9. (This problem is an investigation on a computer.) Create an identity matrix (for example, in Julia or Matlab, `I=eye(5)`) and a permutation vector (example `p=[3, 4, 1, 2, 5]`). Create a permutation matrix by permuting the columns. (In Julia this is I[:,p] and in Matlab, I(:,p),

   Compute matrix powers (`P,P^2,P^3,P^4,...`) Which is the smallest positive power that returns P to the identity? This number is called the *order* of the permutation matrix P.

   Now create a random permutation matrix of size 10 and compute the average order sat by taking 10,000 (or more) samples This can be done in Julia with

```
rand_perm_matrix(n)=eye(n)[:,randperm(n)]
function order(P)
         n=size(P,1)
         k=1
         while (P^k!=eye(n))
           k=k+1
         end
         return(k)
       end
```

For example, type

```
P=rand_perm_matrix(10)
order(P)
```

Finally compute

```
mean( [order(rand_perm_matrix(10)) for i=1:10000])
```

I think the true mean is around 10.75 or so, but your answer will vary.

10. Last year, the fastest computer in the world, according to the biannual linpack benchmark top 500 figures, is an IBM BLUE GENE at Lawrence Livermore Lab with 1,572,864 core processors. It has been clocked at solving a general linear system with $N = 12,681,215$ variables in very impressive time T. A solve takes about $(2/3)N^3$ floating point operations, where N is the number of inputs.

    Next year, the IBM BLUE GENE will be replaced by an Atari Linear Algebra Master 3000 Turbo, which is twice as fast as the BLUE GENE (in that it can execute twice as many floating point operations per second). A group of impatient meteorologists wishes to solve a linear system in time no more than T using the Atari Linear Algebra Master 3000 Turbo. What is the maximum number of inputs N their linear system can have?

18.06 Wisdom. The best use of a computer is not for a one time answer to a problem, but rather to have a laboratory where one can explore and play. You will see our computational problems are not about solving $Ax = b$, but rather about playing with linear algebra ideas designed to tickle the imagination.