18.06 Professor Edelman Quiz 2 November 6, 2013

Your PRINTED name is: _____

Please circle your recitation:

1	T 9	Dan Harris	E17-401G	3-7775	dmh
2	T 10	Dan Harris	E17-401G	3-7775	dmh
3	T 10	Tanya Khovanova	E18-420	4-1459	tanya
4	T 11	Tanya Khovanova	E18-420	4-1459	tanya
5	T 12	Saul Glasman	E18-301H	3-4091	sglasman
6	T 1	Alex Dubbs	32-G580	3-6770	dubbs
7	T 2	Alex Dubbs	32-G580	3-6770	dubbs

Grading

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1 (25 pts.)

Compute the determinant of

a) (10 pts.) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1806 & 1806 & 0 \\ 2013 & 2014 & 2015 \end{bmatrix}$

b) (15 pts.)

The $n \times n$ matrix A_n has ones in every element off the diagonal, and also $a_{11} = 1$ as well. The rest of the diagonal elements are 0: $a_{22} = a_{33} = \ldots = a_{nn} = 0$. For example

$$A_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Write the determinant of A_n in terms of n in simplest form. Argue briefly but convincingly your answer is right.

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2 (30 pts.)

Let $Q = [q_1\ q_2\ q_3]$ be an $m \times 3$ real matrix with $m > 3$ and $Q^TQ = I_3$, the 3×3 identity.

Let $P = QQ^T$.

a) (7 pts.) What are all possible values of $\det(P)$?

b) (7 pts.) What are all the eigenvalues of the $m \times m$ matrix P including multiplicities?

c) (8 pts.) Find one eigenvalue, eigenvector pair of the non-symmetric $m \times m$ matrix $q_1 q_2^T$.

d) (8 pts.) What are the four fundamental subspaces of $M = I - P$ in terms of the column space of P?

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3 (20 pts.)

Let A be a 4×4 general matrix and x a scalar variable. Circle your answers and provide a very brief explanation.

a) (5 pts.) What kind of polynomial in x best describes $\det(A - xI)$?

constant linear quadratic cubic (degree 3) quartic (degree 4)

b) (5 pts.) What kind of polynomial in A_{11} best describes $\det(A - xI)$?

constant linear quadratic cubic (degree 3) quartic (degree 4)

c) (5 pts.) What kind of polynomial in x best describes $\det(xA)$?

constant linear quadratic cubic (degree 3) quartic (degree 4)

d) (5 pts.) What kind of polynomial in x best describes $\det(A(x))$, where

$$A(x) = \begin{bmatrix} xA_{11} & xA_{12} & xA_{13} & xA_{14} \\ A_{21} + x & A_{22} + x & A_{23} + x & A_{24} + x \\ A_{31} - x & A_{32} - x & A_{33} - x & A_{34} - x \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}$$

constant linear quadratic cubic (degree 3) quartic (degree 4)

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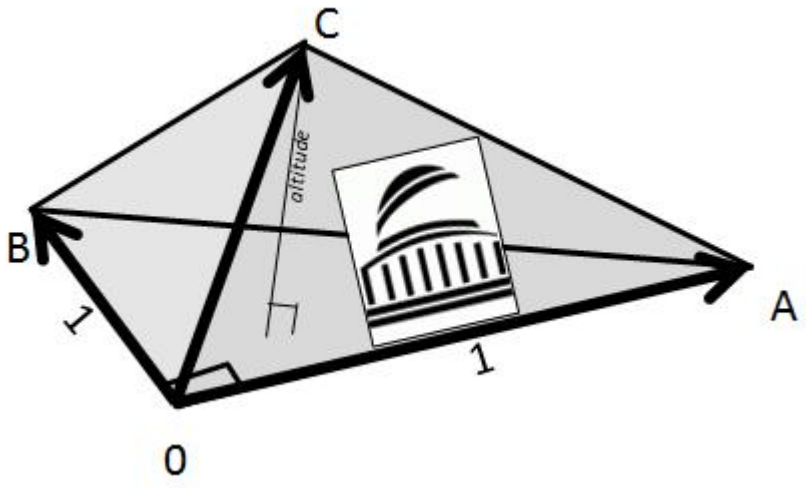

4 (20 pts.)

In R^3 an artist plans an MIT triangular pyramid artwork with one vertex at the origin. The other three vertices are at the tips of vectors A, B and C.

The triangular base of the pyramid $(0, A, B)$ is an isosceles right triangle, The vectors A and B are unit vectors orthogonal to each other.

The other vector C is not in any especially convenient position.

a) (12 pts.) Write an expression for L the length of the altitude of the top of the pyramid to the base in terms of A, B and C.

b) (8 pts.) Write an expression for the volume of the pyramid.