18.06 F13 EXAM 1

SOLUTIONS

1.

a) $$\left(\begin{array}{ccc|c} 2 & 3 & 5 & b_1 \\ 2 & 4 & 5 & b_2 \\ -2 & 0 & -5 & b_3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 3 & 5 & b_1 \\ 0 & 1 & 0 & b_2 - b_1 \\ 0 & 3 & 0 & b_3 + b_1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 2 & 3 & 5 & b_1 \\ 0 & 1 & 0 & b_2 - b_1 \\ 0 & 0 & 0 & b_3 + b_1 - 3(b_2 - b_1) \end{array}\right)$$

$$U = \begin{pmatrix} 2 & 3 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 3 & 1 \end{pmatrix}$$

b) From bottom row of augmented matrix,

$$b_3 + b_1 - 3(b_2 - b_1) = 4b_1 - 3b_2 + b_3 = 0.$$

c) Free column of U is on right, so put a 1 in bottom entry

$$\begin{pmatrix} -5/2 \\ 0 \\ 1 \end{pmatrix}$$

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2. a) A contains columns $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ which are independent

$\therefore$ column space $= \mathbb{R}^2$

b) First subtract top row from bottom row so that we have columns $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 0.451 & 0.3 & 0 & 0.2 & 1 & -0.1 \\ 0.222 & 0.4 & 1 & 0.3 & 0 & -0.2 \end{pmatrix}$$

Four independent solutions:

$$\begin{pmatrix} 1 \\ 0 \\ -0.222 \\ 0 \\ -0.451 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ -0.4 \\ 0 \\ -0.3 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ -0.3 \\ 1 \\ -0.2 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0.2 \\ 0 \\ 0.1 \\ 1 \end{pmatrix}$$

(corresponding to the free columns 1, 2, 4, 6).

3\. a) If the column space is a plane, the nullspace is a line.

If the column space is a line, the nullspace is a plane.

If the column space is $\mathbb{R}^3$, the nullspace is zero.

If the column space is zero, the nullspace is $\mathbb{R}^3$.

b) dim(column space) + dim(nullspace) = 7

If column space = nullspace then they both have dimension $7/2$, but dimension cannot be a fraction.

So the matrix A cannot exist.

4.

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & x \end{pmatrix}$$

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5. Gauss-Jordan for nonsingular square matrix:
In each of the n columns, we use the pivot to reduce each of the other $n-1$ entries to 0.
Therefore we use $n(n-1)$ operations.