18.06 Professor Edelman Quiz 1 October 4, 2013

Grading

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Your PRINTED name is: ____________________

Please circle your recitation:

1	T 9	2-132	Dan Harris	E17-401G	3-7775	`dmh`
2	T 10	2-132	Dan Harris	E17-401G	3-7775	`dmh`
3	T 10	2-146	Tanya Khovanova	E18-420	4-1459	`tanya`
4	T 11	2-132	Tanya Khovanova	E18-420	4-1459	`tanya`
5	T 12	2-132	Saul Glasman	E18-301H	3-4091	`sglasman`
6	T 1	2-132	Alex Dubbs	32-G580	3-6770	`dubbs`
7	T 2	2-132	Alex Dubbs	32-G580	3-6770	`dubbs`

1 (30 pts.)

Consider the matrix $A = \begin{bmatrix} 2 & 3 & 5 \\ 2 & 4 & 5 \\ -2 & 0 & -5 \end{bmatrix}$ and the general right hand side $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

a) (18 pts.) Reduce A to an upper triangular matrix U and carry out the same steps on the right side b by working with the augmented matrix $[A\ b]$. Factor the 3 by 3 matrix A into $LU =$(lower triangular)(upper triangular).

b) (6 pts.) Describe the column space of A exactly through a condition on b.

c) (6 pts.) What are the special solutions to $Ax = 0$?

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2 (25 pts.)

Consider the matrix $A = \begin{bmatrix} 0.451 & 0.3 & 0 & 0.2 & 1 & -.1 \\ 0.673 & 0.7 & 1 & 0.5 & 1 & -.3 \end{bmatrix}$. (Big Hint: The questions asked here can all be readily done with mental arithmetic if you reorder your world view.)

a) (5 pts.) What is the column space of A? (Explain briefly.)

c) (20 pts.) Write down four independent solutions to $Ax = 0$.

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3 (15 pts.)

a) (4 pts.) Complete these sentences appropriately for a 3×3 matrix A.

If the column space is a plane, the nullspace is a ____________________.

If the column space is a line, the nullspace is a ____________________.

If the column space is all of R^3, the nullspace ____________________.

If the column space is the zero vector, the nullspace ____________________.

b) (11 pts.) Find a 7×7 matrix A whose column space equals its nullspace, or argue briefly it can not exist. (Hint: part 3a might provide a clue.)

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4 (15 pts.)

The vector space S consists of 2×2 matrices whose entries are linear functions of the symbol x. For example, $\begin{bmatrix} x & 2-x \\ 1+x & 4+10x \end{bmatrix}$ is one member of S, and the general form of a member of S is

$$A = \begin{bmatrix} a+bx & e+fx \\ c+dx & g+hx \end{bmatrix}.$$

Write down a basis for S.

5 (15 pts.)

An elimination step (a multiple of one row subtracted from another row) may be written in Julia as

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A[j,:]-=m*A[i,:]
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where we assume $i \neq j$.

The same row operation in matrix form is expressed in linear algebra by "replace A with EA," where E is the matrix formed from the identity with $-m$ in the (j, i) entry.

If Gauss-Jordan is performed on an n by n non-singular matrix A, augmented with I, provide an exact count in terms of n of the general number of required elimination steps. (Hint: we are counting in units of row operations, not elemental operations; the exact answer has the form $an^2 + bn + c$). We want the exact answer and a very short reason why.)

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