18.06 Professor Edelman Quiz 3 December 5, 2011

Grading

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Your PRINTED name is: ____________________

Please circle your recitation:

1	T 9	2-132	Kestutis Cesnavicius	2-089	2-1195	kestutis
2	T 10	2-132	Niels Moeller	2-588	3-4110	moller
3	T 10	2-146	Kestutis Cesnavicius	2-089	2-1195	kestutis
4	T 11	2-132	Niels Moeller	2-588	3-4110	moller
5	T 12	2-132	Yan Zhang	2-487	3-4083	yanzhang
6	T 1	2-132	Taedong Yun	2-342	3-7578	tedyun

1 (24 pts.)

Let $A = \begin{pmatrix} .5 & 0 & 0 \\ .5 & .9 & 0 \\ 0 & .1 & 1 \end{pmatrix}$.

1. (4 pts) True or False: The matrix A is Markov.

2. (6 pts) Find a vector $x \neq 0$ and a scalar λ such that $A^T x = \lambda x$.

3. (4 pts) True or False: The matrix A is diagonalizable. (Explain briefly.)

4. (4 pts) True or False: One singular value of A is $\sigma = 0$. (Explain briefly.)

5. (6 pts) Find the three diagonal entries of e^{At} as functions of t.

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2 (30 pts.)

1. (5 pts) An orthogonal matrix Q satisfies $Q^TQ = QQ^T = I$. What are the n singular values of Q?

2. (10 pts) Let $A = \begin{pmatrix} 1 & & \\ & -2 & \\ & & 3 \end{pmatrix}$. Find an SVD, meaning $A = U\Sigma V^T$, where U and V are orthogonal, and $\Sigma = \begin{pmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{pmatrix}$ is diagonal with $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0$. (Be sure that the factorization is correct and satisifies all stated requirements.)

3. (15 pts) The 2×2 matrix $A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T$, where $\sigma_1 > \sigma_2 > 0$ and both u_1, u_2 and v_1, v_2 are orthonormal bases for R^2.

The set of all vectors x with $\|x\| = 1$ describes a circle in the plane. What shape best describes the set of all vectors Ax with $\|x\| = 1$? Draw a general picture of that set labeling all the relevant quantities σ_1, σ_2, u_1, u_2 and v_1, v_2. (Hint: Why are v_1, v_2 relevant and u_1, u_2 not relevant?)

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3 (16 pts.)

1. (6 pts) Let $x \neq 0$ be a vector in R^3. How many eigenvalues of $A = xx^T$ are positive? zero? negative? (Explain your answer.)

2. (6 pts) a) What are the possible eigenvalues of a projection matrix?

   b) True or False: every projection matrix is diagonalizable.

3. (4 pts) True or False: If every eigenvalue of A is 0, then A is similar to the zero matrix.

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4 (30 pts.)

Consider the matrix $A = \begin{pmatrix} x & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ with parameter x in the (1,1) position.

1. (10 pts) Specify all numbers x, if any, for which A is positive definite. (Explain briefly.)

2. (10 pts) Specify all numbers x, if any, for which e^A is positive definite. (Explain briefly.)

3. (10 pts) Find an x, if any, for which $4I - A$ is positive definite. (Explain briefly.)

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