18.06 Professor Edelman Quiz 2 November 4, 2011

Grading
1
2
3
4

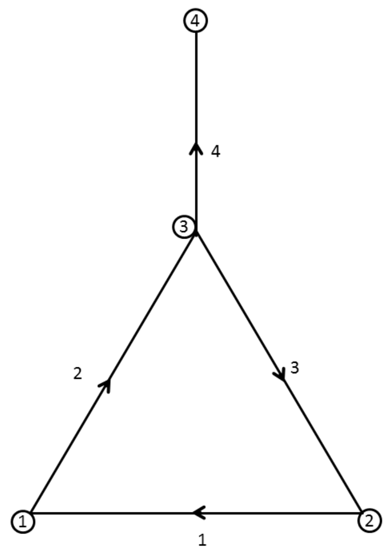
Your PRINTED name is: ____________________

Please circle your recitation:

1	T 9	2-132	Kestutis Cesnavicius	2-089	2-1195	`kestutis`
2	T 10	2-132	Niels Moeller	2-588	3-4110	`moller`
3	T 10	2-146	Kestutis Cesnavicius	2-089	2-1195	`kestutis`
4	T 11	2-132	Niels Moeller	2-588	3-4110	`moller`
5	T 12	2-132	Yan Zhang	2-487	3-4083	`yanzhang`
6	T 1	2-132	Taedong Yun	2-342	3-7578	`tedyun`

1 (30 pts.)

Consider the directed graph with four vertices and four edges pictured below:



1. (7 pts) The 4×4 incidence matrix of this directed graph is:

2. (7 pts) Find the determinant of the incidence matrix. (The easy way or the hard way)

3. (8 pts) Find a basis for the column space of the incidence matrix (Note this can be done with or without the answer in part 1.)

4. (8 pts) Consider whether or not it is possible to have an incidence matrix for a graph with n nodes and n edges that is invertible. If it is possible, draw the directed graph, if not possible, argue briefly why not.

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2 (20 pts.)

1. (10 pts) Project the function $\sin(x) + \cos(x)$ defined on the interval $[0, 2\pi]$ onto the three dimensional space of functions spanned by $\cos x$, $\cos 2x$, and $\cos 3x$. Express the (hint: very simple) answer in simplest form. Briefly explain your answer.

2. (10 pts) Write down all $n \times n$ permutation matrices that are also projection matrices.

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3 (15 pts.)

3. (10 pts) What are all possible values for the determinant of a projection matrix? (Please explain briefly.)

4. (5 pts) What are all possible values for the determinant of a permutation matrix? (Please explain briefly.)

4 (35 pts.)

1. (20 pts) The matrix A is 2000×2000 and $A^T A = I$. Let v be the vector $[1, 2, 3, \ldots, 2000]^T$. Let v_1 be the projection of v onto the space spanned by the first 1000 columns of A. Let v_2 be the projection of v onto the space spanned by the remaining 1000 columns of A. What is $v_1 + v_2$ in simplest form? Why? Give an example of a 2000×2000 A, where $A^T A \neq I$, and where $v_1 + v_2$ gives a different answer.

2. (15 pts) In a matrix A, (which may not be invertible) the cofactors from the first row are $C_{11}, C_{12}, \ldots, C_{1n.}$. Prove that the vector $C = (C_{11}, C_{12}, \ldots, C_{1n.})$ is orthogonal to every row of A from row 2 to row n. Hint: the dot product of C with row i $(i = 2, \ldots, n)$ is the determinant of what matrix?

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