18.06 (Fall '11) Problem Set 9

This problem set is due Thursday, November 17, 2011 at 4pm. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary("filename")` will start a transcript session, `diary off` will end one.)

1. Do Problem 5 from 8.3. Surprising?
2. Do Problem 12 from 8.3.
3. Do Problem 3 from 6.3.
4. Do Problem 4 from 6.3.
5. Do Problem 5 from 6.3.
6. Do Problem 8 from 6.4.
7. Do problem 10 from 6.4.
8. Do Problem 20 from 6.4 (in some sense, this is the cornerstone of quantum mechanics).
9. Let $J = \begin{pmatrix} 0.4 & 0.2 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.4 & 0.1 \end{pmatrix}$. This Markov matrix describes surfing behavior in a universe with only four web pages. The $(i.j)$ entry is the probablity that your next browser experience is site i, given that you are currently on j. Note that you can return to the same site again. Using a computer, rank the four web pages in order using the steady state. (you can play with different numbers and consider whether this "pagerank" matches your intuition).
10. Let A be a fixed 2×2 matrix. Show that all the solutions u to $u' = Au$ form a subspace of the (very big) vector space W of functions $\begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}$ (you do not need to show that this space of functions is a vector space, but it is good practice to convince yourself). Let V be the subspace of W where each of the $f_i(t)$ above are linear combinations of exponential functions. Give an example of an A where the solutions to $u' = Au$ form a subspace of V (hint: this should be true for most A).

18.06 Wisdom. The concept of a steady state (or more generally, an eigenstate) is powerful. On the macro-side of life you see populations and wealth, and on the micro-side of life you see energy levels and temperature. A lot of the states of the world are really steady states because they are results of little things balancing out over a long time (to steal an economics analogy, you can think of the "invisible hand" of big matrix powers). This explains why people look for equilibria and eigenstates everywhere from economics to quantum mechanics.