18.06 (Fall '11) Problem Set 8

This problem set is due Thursday, November 10, 2011 at 4pm. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary("filename")` will start a transcript session, `diary off` will end one.)

1. What are the possible eigenvalues of a projection matrix? (Hint: if $P^2 = P$ and v is an eigenvector, look at P^2v and Pv). Show that the values you give are all possible.

2. What are the possible real eigenvalues of a 4 by 4 permutation matrix? (Hint: consider such a matrix P and powers $I, P, P^2, P^3, \ldots$. Show it eventually has to repeat). You do not need to show the values you give are all possible, but you still must get the correct range to achieve full credit.

3. (Do this problem for both the permutations $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$). Given the matrix M, find the characteristic polynomial $f(x) = det(M - xI)$. In both cases, try to "apply $f(x)$ to M," in the sense that if $f(x) = ax^2 + bx + c$, compute $f(M) = aM^2 + bM + cI$. What happens? (This is called the Cayley-Hamilton Theorem and is always true, though you do not need to prove it. You can use it to prove a lot of cool things.)

4. Do problem 6 from 6.1.

5. Do problem 11 from 6.1.

6. Do problem 19 from 6.1.

7. Do problem 4 from 6.2.

8. Do Problem 10 from 6.2 (you must use linear algebra somehow).

9. Do Problems 11 and 12 from 6.2.

10. Do Problem 7 from 8.3 (including the challenge problem!).

18.06 Wisdom. A lot of math (at least 18.06) is about setting up equations and then doing the same things to both sides in skillful ways.