18.06 (Fall '11) Problem Set 5

This problem set is due Thursday, October 20, 2011 at 4pm. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary("filename")` will start a transcript session, `diary off` will end one.)

1. Take two connected graphs A and B with a and b vertices respectively. Let their union be C (i.e. a big graph with A and B as two disjoint parts).

   (a) What is the rank of C's incidence matrix?

   (b) What is its nullspace?

2. Free points!!!

3. Do problem 21 from 4.1.

4. Do problem 28 from 4.1.

5. Do problem 11(a) from 4.2. Check the "it should..." part!

6. Do problem 17 from 4.2. Briefly explain your answer.

7. Do problem 22 from 4.2.

8. On a computer:

   (a) project a vector b onto the column space of a matrix A with independent columns (in MATLAB, try `A=randn(3,2)`. When making the projection matrix, keep in mind that `A'` is the transpose and `\inv(A)` is the inverse). Project the result onto the same A. Explain the results.

   (b) Construct an A where the columns are not independent. Now try to make a projection matrix. What happens? Explain why MATLAB (or whatever software you are using) would do this.

   (c) Let the projection matrix you get in the first part be P. Add $P+P^2+\cdots+P^{100}$. (in MATLAB this would be the following code: `Z=0; for i=1:100, Z=Z+P^i; end`. Compare the answer to P. Explain what you see.

9. Do problem 6 from 4.3.

10. Do problem 12 from 4.3.

18.06 Wisdom. The most brutal but also the most helpful way to prepare for an exam is this: flip to a problem that you've already solved (in homework). Do that problem with no help whatsoever. You'll be disgusted at how little you've remembered and then that problem will be burned into your head (at least for the exam). If you can't do it - well, you now know what to study.