18.06 (Fall '11) Problem Set 3

This problem set is due Thursday, September 29, 2011 at 4pm. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary("filename")` will start a transcript session, `diary off` will end one.)

1. ("these matrices" refer to the *original* matrices in Problem 3.2.1, not reduced forms)

   (a) Do problem 1 from 3.2.

   (b) Compute the column space and the null space for these matrices.

   (c) Compute the rank of these matrices.

   (d) For both of these matrices M, solve $Mx = b$, where $b = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$.

2. Do problem 9 from 3.2.

3. What is the sum of the number of free and pivot variables for a $m * n$ matrix in general? The answer should just depend on m and n.

4. Do problem 1 from 3.3.

5. Do problem 8 from 3.3.

6. Do problem 13 from 3.4.

7. Fill a n by n matrix with random bits (i.e. either 1 or 0) and caluclate its rank. In MATLAB this is `r = rank(randi([0 1],n))`. Do this many times for $n = 1, 5, 10$ and 15. What do you observe for these 4 situations? (Taking an average over the many times would be best, but you can also do this by just looking. You don't have to show each of the "many times" in your print out)

8. *You do not need to touch MATLAB, or even the computer, for this problem.* Here is what happens when one uses MATLAB 's rank command:

   ```
   >> e=1e-15; a=[1+e 1;1  1]; rank(a)
   ans =
         1
   ```

   (the first command just means that $e = 10^{-15}$) Show that this is not mathematically correct. Why do you think MATLAB produces this answer? (No need to read MATLAB documentations - a couple of sentences with a reasonable guess would suffice)

9. Do problem 16 from 3.5.

10. Do Problem 26 from 3.5.

18.06 Wisdom. For many people, the most important thing about a matrix is whether it is full rank or not. Ask yourself what this means for you now. Later, you'll realize this is an important and sometimes subtle question.