18.06 (Fall '11) Problem Set 2

This problem set is due Thursday, September 22 2011 at 4pm, in room 2−114. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary("filename")` will start a transcript session, `diary off` will end one.)

1. Do Problem 7 from 2.6.
2. Do Problem 15 from 2.6. .
3. Do these problems about permutations.
   (a) Do Problem 8 from 2.7.
   (b) For each permutation matrix of size 3, tell me what each one does to the column vector $v = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. For example, the identity matrix of size 3 sends v to itself, and so corresponds to $v = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.
   (c) For 3 matrices of different $n > 1$, take a permutation matrix of size n and take their $(n!)$-th powers. What do you get? Think about why. The proof is outside the scope of this course (in MATLAB, we can create a random permutation matrix with `e=eye(n); a=e(randperm(n),:)`. To evaluate a factorial, use `factorial(n)`).
4. Do Problem 19 from 2.7.
5. Do Problem 22 from 2.7.
6. Suppose a portfolio consists of a credit account, a checkings account, and a savings account, with the premise that for all these accounts you can be in the negatives (overdrawing, etc.) with no limit in either direction. Convince the grader that the set of portfolios has the structure of a vector space. How many dimensions are there? What do they mean? What does addition/subtraction of two vectors mean in this space? What does multiplication of a vector by a real number mean in this space?
7. Do Problem 14 from 3.1.
8. Do Problem 27 from 3.1.
9. In this problem, we'll investigate how long it takes for computers to do matrix multiplications.
   (a) On a computer, time the following operations for random matrices of size 100, 200, and 800, averaged over say 50 trials: a) matrix multiplication, b) matrix addition, c) solving $Ax = b$. With MATLAB, your code for multiplication may look like this after setting an n:

```
t=50
v=zeros(t,1)
for i=1:t
a=rand(n); b=rand(n);
tic, a*b; v(i)=toc;
end
mean(v)
```

(b) Now, compute the *rate* of computation (single-number additions and multiplications per second) for each of these three operations on these various n. Recall that matrix multiplication, matrix addition, and equation solution require roughly $2n^3, n^2$, and $(2/3)n^3$ single-number additions and multiplications, respectively (technical fact: the rate may converge to different numbers for reasons beyond the scope of this class, like memory traffic and cache misses).

10. Create random 2x2x2 3-dimensional arrays (not matrices, which are 2-dimensional arrays!) of numbers (in MATLAB, `a=rand(2,2,2);`).

    (a) Create two such 3-dimensional arrays. Add them. Multiply them by constants. Get a feel for them. Question: do these arrays form a vector space?

    (b) Suppose we choose a different set of (m, n, p) besides $(2, 2, 2)$. Are the `rand(m,n,p)`'s a vector space? (avoid any of these being 1 since MATLAB does something funny)

    (c) Can I add a `rand(2,3,5)` and a `rand(4,7,8)`?

    (d) True or false: the collection of ALL 3-dimensional arrays form a vector space.

18.06 Wisdom. Theory is neither as important as mathematicians tell you nor as useless as engineers tell you.