18.06 Problem Set 5

Due Thursday, 14 October 2010 at 4pm in the undergrad math office. Please note that the problems from the textbook are out of the 4th edition: make sure to check that you are doing the correct problems. For MATLAB problems, please include a printout of your code with your problem set. You can type `diary(''filename'')` at the beginning of your session to save a transcript, and `diary` off when you are done.
Each Problem worth 10 points.

1. Do problem 14 from section 8.2.

2. Give all solutions of the system

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 8 \\ 5 \\ 2 \end{pmatrix}$$

3. Consider the matrix A from the previous excercise. What is the rank of A? We know that $r(AB) \leq r(A)$ and $r(AB) \leq r(B)$, thus $r(AB) \leq \min\{r(A), r(B)\}$. What can be the rank of B if $r(AB) = \min\{r(A), r(B)\}$. Give examples for B for all possible ranks. If the above equality cannot hold for some rank of B prove it.

4. Do problem 11 from section 4.1.

5. Do problem 17 from section 4.1.

6. Call a square matrix A orthogonal if all of its columns are of length 1, and are orthogonal to each other.
   (a) What is the product AA^T?
   (b) Prove that if A is orthogonal then so is A^T.
   (c) Suppose that A and B are orthogonal matrices. Prove that AB is orthogonal too.

7. Do problem 5 from section 4.2.

8. Do problem 14 from section 4.2.

9. Do problem 17 from section 4.2.

10. Write a program in MATLAB or your favorite language, to Project a vector b onto the column space of a matrix A with independent columns. Try perhaps A=randn(3,2). What happens numerically if b is in the nullspace of this A? What happens when you run the program on a matrix A where the columns are not independent? What happens if one column of A is nearly dependent on the others? (Add .0001 to a linear combination of the others, for example.)