18.06 Professor Edelman Final Exam December 15, 2010

Your PRINTED name is: ____________________

Please circle your recitation:

R01	T 9	2-132	S. Kleiman	2-278	3-4996	kleiman
R02	T 10	2-132	S. Kleiman	2-278	3-4996	kleiman
R03	T 11	2-132	S. Sam	2-487	3-7826	ssam
R04	T 12	2-132	Y. Zhang	2-487	3-7826	yanzhang
R05	T 1	2-132	V. Vertesi	2-233	3-2689	18.06
R06	T 2	2-131	V. Vertesi	2-233	3-2689	18.06

Grading

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1 (16 pts.)

a. (8 pts) Give bases for each of the four fundamental subspaces of $A = \begin{bmatrix} 1 & 0 & \pi & e \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b. (8 pts) Give bases for each of the four fundamental subspaces of

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 2 & & \\ & & 3 & \\ & & & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(Each of the three matrices in the above product has orthogonal columns.)

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2 (14 pts.)

Let P_1 be the projection matrix onto the line through $(1, 1, 0)$ and P_2 is the projection onto the line through $(0, 1, 1)$.

(a) (4 pts) What are the eigenvalues of P_1?

(b) (10 pts) Compute $P = P_2 P_1$. (Careful, the answer is not 0)

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3 (10 pts.)

The nullspace of the matrix A is exactly the multiples of $(1, 1, 1, 1, 1)$.

(a) (2 pts.) How many columns are in A?

(b) (3 pts.) What is the rank of A?

(c) (5 pts.) Construct a 5×5 matrix A with exactly this nullspace.

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4 (15 pts.)

Find the solution to

$$\frac{du}{dt} = -\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} u$$

starting with $u(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$.

(Note the minus sign.)

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5 (10 pts.)

The 3×3 matrix A satisfies $\det(tI - A) = (t-2)^3$.

(a) (2pts) What is the determinant of A?

(b) (8pts) Describe all possible Jordan normal forms for A.

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6 (7 pts.)

The matrix $A = \begin{bmatrix} 1 & 0 \\ C & 1 \end{bmatrix}$

(a) (2 pts) What are the eigenvalues of A?

(b) (5 pts) Suppose σ_1 and σ_2 are the two singular values of A. What is $\sigma_1^2 + \sigma_2^2$?

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7 (8 pts.)

For each transformation below, say whether it is linear or nonlinear, and briefly explain why.

(a) (2 pts) $T(v) = v/\|v\|$

(b) (2 pts) $T(v) = v_1 + v_2 + v_3$

(c) (2 pts) $T(v) =$ smallest component of v

(d) (2 pts) $T(v) = 0$

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8 (10 pts.)

V is the vector space of (at most) quadratic polynomials with basis $v_1 = 1, v_2 = (x-1), v_3 = (x-1)^2$. W is the same vector space, but we will use the basis $w_1, w_2, w_3 = 1, x, x^2$.

(a) (5 pts) Suppose $T(p(x)) = p(x+1)$. What is the 3×3 matrix for T from V to W in the indicated bases?

(b) (5 pts) Suppose $T(p(x)) = p(x)$. What is the 3×3 matrix for T from V to W in the indicated bases?

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9 (10 pts.)

In all of the following we are looking for a real 2×2 matrix or a simple and clear reason that one can not exist.

Please remember we are asking for a real 2×2 matrix.

(a) (2 pts) A with determinant -1 and singular values 1 and 1.

(b) (2 pts) A with eigenvalues 1 and 1 and singular values 1 and 0.

(c) (2 pts) A with eigenvalues 0 and 0 and singular values 0 and 1

(d) (2 pts) A with rank $r = 1$ and determinant 1

(e) (2 pts) A with complex eigenvalues and determinant 1

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