18.06 Professor Edelman Quiz 3 December 1, 2010

Grading

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2

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Your PRINTED name is: ____________________

Please circle your recitation:

R01	T 9	2-132	S. Kleiman	2-278	3-4996	kleiman
R02	T 10	2-132	S. Kleiman	2-278	3-4996	kleiman
R03	T 11	2-132	S. Sam	2-487	3-7826	ssam
R04	T 12	2-132	Y. Zhang	2-487	3-7826	yanzhang
R05	T 1	2-132	V. Vertesi	2-233	3-2689	18.06
R06	T 2	2-131	V. Vertesi	2-233	3-2689	18.06

1 (30 pts.)

In the following six problems produce a real 2×2 matrix with the desired properties, or argue concisely, simply, and convincingly that no example can exist.

(a) (5 pts.) A 2×2 symmetric, positive definite, Markov Matrix.

(b) (5 pts.) A 2×2 symmetric, negative definite (i.e., negative eigenvalues), Markov Matrix.

(c) (5 pts.) A 2×2 symmetric, Markov Matrix with one positive and one negative eigenvalue.

(d) (5 pts.) A 2×2 matrix $\neq 3I$ whose only eigenvalue is the double eigenvalue 3.

(e) (5 pts.) A 2×2 **symmetric** matrix $\neq 3I$ whose only eigenvalue is the double eigenvalue 3.
(Note the word "symmetric" in problem (e).)

(f) (5 pts.) A 2×2 non-symmetric matrix with eigenvalues 1 and -1.

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2 (35 pts.)

Let

$$A = -\begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$$

(Note the minus sign in the definition of A.)

(a) (15 pts.) Write down a valid SVD for A. (No partial credit for this one so be careful.)

(b) (20 pts.) The 4×4 matrix $e^{At} = I + f(t)A$. Find the scalar function $f(t)$ in simplest possible form. (Hint: the power series is one way; eigendecomposition is another.)

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3 (35 pts.)

(a) (15 pts.) The matrix A has independent columns. The matrix C is square, diagonal, and has positive entries. Why is the matrix $K = A^T CA$ positive definite? You can use any of the basic tests for positive definiteness.

(b) (20 pts.) If a diagonalizable matrix A has orthonormal eigenvectors and real eigenvalues must it be symmetric? (Briefly why or give a counterexample)

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