1 (30 pts.)

(a) (25 pts.) Compute the determinant (as a function of x) of the 4×4 matrix

$$A = \begin{bmatrix} x & x & x & x \\ x & x & 0 & 0 \\ x & 0 & x & x \\ x & 0 & x & 1 \end{bmatrix}$$

(Note that all the entries x in the matrix represent the same number.)

(b) (5 pts.) Find all values of x for which A is singular.

2 (35 pts.)

Let P_1 be the projection matrix onto the line through $(1, 1, 0)$ and P_2 is the projection matrix onto the line through $(0, 0, 1)$.

(a) (15 pts.) Compute $P = P_2P_1$. Note that there is a harder way and an easier way to perform this computation. Either way is valid. (The easier way uses associativity of matrix multiplication. Always a useful trick.)

(b) (5 pts.) Is $P = P_2P_1$ a projection matrix? (Explain simply.)

(c) (15 pts.) What are the four fundamental subspaces associated with P?

3 (35 pts.)

(a) (10 pts.) Perform Gram Schmidt on the two vectors $u = (1,1,1,1)$ and $t = (t_1, t_2, t_3, t_4)$. The answer should be in the form q_1 and q_2, an orthonormal pair of vectors. You may wish to use the notation $\bar{t}$ for the mean of t, and $\|t - \bar{t}u\|$ for the norm of $t - \bar{t}u$.

(b) (10 pts.) Write a "QR" decomposition of $[u\ t]$, i.e. find a 4×2 matrix Q, and a 2×2 matrix R such $[u\ t] = QR$, where $Q^T Q = I$, and $R_{2,1} = 0$.

(c)(5 pts.) When is R singular?

(d)(10 pts) Use the QR decomposition of $A = [u\ t]$ (and not the normal equations with $A^T A$), to compute the slope of the best fit line $C + Dt$ to the data (t_i, b_i) for $i = 1, 2, 3, 4$. (In other words, compute a simple expression for D.)

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