

**Grading**

**1**

Your PRINTED name is: \_\_\_\_\_

**2**

**3**

**4**

Please circle your recitation:

\_\_\_\_\_

R01	T 9	2-132	S. Kleiman	2-278	3-4996	kleiman
R02	T 10	2-132	S. Kleiman	2-278	3-4996	kleiman
R03	T 11	2-132	S. Sam	2-487	3-7826	ssam
R04	T 12	2-132	Y. Zhang	2-487	3-7826	yanzhang
R05	T 1	2-132	V. Vertesi	2-233	3-2689	18.06
R06	T 2	2-131	V. Vertesi	2-233	3-2689	18.06

**1 (30 pts.)**

Start with the 3 by 4 matrix:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \end{bmatrix}.$$

- (a) (10 pts.) What are all the special solutions to  $Ax = 0$ , and describe the nullspace of  $A$ .

(b) (10 pts.) What is the rank of  $A$ , and describe the column space of  $A$ .

(c) (5 pts.) Find all solutions to  $Ax = \begin{bmatrix} 0 \\ 6 \\ 12 \end{bmatrix}$ .

(d) (5 pts.) Can  $A$  be written as  $A = uv^T$  for some vectors  $u$  and  $v$ ? If so what are these vectors, or if not, why not?

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**2 (30 pts.)** Consider the matrix

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix},$$

where  $ps = rq$  and  $pr \neq 0$ .

(a) (5 pts.) Describe simply and clearly the column space of  $A$ .

(b) (10 pts.) Write as simply as possible the special solution(s) to  $Ax = 0$ , if any.

(c) (5 pts.) What are all the solutions to  $Ax = 0$ ?

(d) (10 pts.) Write  $A$  as simply as possible in row reduced echelon form.

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**3 (20 pts.)**

(In the questions below, you can choose any  $n$  that works for an example, or prove that for all  $n$ , there is no example.)

(a) (10 pts.) Can you find independent vectors  $v, w, x$  and  $y$  in some space  $R^n$  and where  $A = vw^T + xy^T$  is invertible? or prove that no such example exists?

(b) (10 pts.) Can you find vectors  $v, w, x$  and  $y$  that span some space  $R^n$  and where  $A = vw^T + xy^T$  is invertible? or prove that no such example exists?



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**4 (20 pts.)**

Write an informal computer program to calculate  $xx^T x$ , for any  $n \times 1$  column vector  $x$ . The program should only use about  $3n$  operations and no more than about  $2n$  numbers in memory. You can write the program in MATLAB or your favorite language. It is not important that you remember exact syntax, but it is important that your operations are clear and unambiguous.

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