18.06 Problem Set 6

Due Wednesday, 4 November 2008 at 4 pm in 2-106.

Please note that the book problems listed below are out of the 4th edition. Please make sure to check that you are doing the correct problems.

Problem 1: Do problem 39 from section 5.3.

Problem 2: Do problems 6 from section 6.1.

Problem 3: Problem 19 section 6.1.

Problem 4: Problem 9 section 6.2.

Problem 5: Do problem 11 in section 6.2.

Problem 6: Do problem 12 in section 6.2.

Problem 7: Let Q be an n by n orthogonal matrix. Let A, B, and C be n by n matrices.

(a) Show that $\det(QAQ^T) = \det(A)$.

(b) The trace of A is the sum of the diagonal entries. $\mathrm{tr}A = \Sigma_{i=1}^{n} a_{ii}$. Show that $\mathrm{tr}(BC) = \mathrm{tr}(CB)$.

(c) Use the result of part (b) to show that $\mathrm{tr}(QAQ^T) = \mathrm{tr}(A)$.

(d) Consider the matrix $A - \lambda I$. Use the big determinant formula to show that $\det(A - \lambda I)$ is a polynomial of degree n.

(e) So now we have

$$\det(A - \lambda I) = \Sigma_{i=0}^{n} c_i \lambda^i,$$

where c_i just denotes the coefficient of the term λ^i in this polynomial. In the case that the dimension of A is 2 by 2, identify the coefficients of this polynomial in terms of trace and determinant.

(d) Show that each coefficient c_i is invariant in the sense that, given orthogonal matrix Q:

$$\det(QAQ^T - \lambda I) = \det(A - \lambda I).$$