18.06 Problem Set 3

Due Wednesday, 30 September 2008 at 4 pm in 2-106.

Please note that the book problems listed below are out of the 4th edition. Please make sure to check that you are doing the correct problems.

Problem 1: Do problem 12 from section 3.3.

Problem 2: Do problem 21 from section 3.4.

Problem 3: When we perform elimination via row operations on a matrix A to end up with an upper triangular matrix U, we know this operation can be expressed as multiplication by an elimination matrix E so that $EA = U$. Why does E change the column space of A but not the nullspace? Give an example of A such that EA has a different column space. Explain why the nullspace of A and U are the same.

Problem 4: Suppose that a is a real, nonzero number. Consider the matrix

$$\begin{pmatrix} a & b & c & c \\ a & a & b & c \\ a & a & a & b \\ a & a & a & a \end{pmatrix}.$$

Find the row reduced echelon form and the nullspace of A when:

(a) $a \neq b$

(b) $a = b$ and $b \neq c$

(c) $a = b = c$.

Problem 5: (MATLAB problem!)

(a) Define a 4 by 4 random matrix A and column vector b:
`A= rand(4); b = [1;1;1;1];` Solve the equation Ay=b by typing:
`y = A \ b` Type: `A*y - b` What do you get?

(b) Write a program to define a matrix A such that $A_{i,j} = e^{15(i+j)}$. To do this we write:

```
for i=1:4
    for j=1:4
        A(i,j)=exp(15*(i+j));
    end
end


A=A.*rand(4);

b=[1;1;1;1];
```

The command A.$*$ rand(4) multiplies each entry in A termwise by a random number. We do this so that our matrix A is not so singluar. Now, type: `x = A \ b`. What is $Ax - b$?

(c) In part (b), why is $Ax - b$ so far from 0? Notice that $Ax - b$ should be equal to 0, because x is supposed to be a solution to the equation $Ax = b$. Hint: In essence, imagine that to solve for x, MATLAB is row reducing. Note that in MATLAB $10^{-17} = 0$ because MATLAB only sees 16 significant figures. (Type: `format long` and look at 10^{-16} and compare to 10^{-17}.)

Problem 6: Do problem 35 from section 3.2 in the book.

Problem 7:

1. Suppose A is a matrix such that $A^2 = A$. True or False
   (a) $N(A)$ is a subspace of $C(A)$.
   (b) $C(A)$ is a subspace of $N(A)$.
   (c) The only vector common to both spaces is 0.
2. Suppose A is a matrix such that $A^2 = 0$. True or False
   (a) $N(A)$ is a subspace of $C(A)$.
   (b) $C(A)$ is a subspace of $N(A)$.
   (c) The only vector common to both spaces is 0.

Be sure to justify your answers and explain why each option is true, or give an example of how it fails.