QUIZ 1 ANSWERS

1. $\begin{pmatrix} 1 & 0 \\ 4 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

a). (12 points) We reduce: $\begin{pmatrix} 1 & 0 & b_1 \\ 4 & 1 & b_2 \\ 2 & -1 & b_3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 - 4b_1 \\ 0 & -1 & b_3 - 2b_1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 - 4b_1 \\ 0 & 0 & b_3 + b_2 - 6b_1 \end{pmatrix}$.

So the equation becomes:

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 - 4b_1 \\ b_3 + b_2 - 6b_1 \end{pmatrix}$.

b) (6 points) Only when $b_3 + b_2 - 6b_1 = 0$.

2. a) (8 points) Solutions to A are length 3 column vectors, so A has three columns.

b) (8 points) Any number. In fact consider $\begin{pmatrix} 1 & -c & -d \end{pmatrix}$. This kills both of the given vectors. Then, feel free to add any number of rows of zero's below it.

c) (8 points) We assume that the matrix is not zero as we are given that these are the only special solutions. To find the rank, we note that each row of A must be in the subspace of $\mathbb{R}^3$ which is orthogonal to $\begin{pmatrix} c & 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} d & 0 & 1 \end{pmatrix}$, but since these guys span a plane, this subspace is a line. Thus the rows are all linearly dependant, and so the rank is one.

3. a) (20 points) False. To get A to its reduced form, you need to multiply on the left by some elimination matrix E; so $Rx = Eb$ is correct, but there is no reason for $Eb - b$ to be in the nullspace of A(you can use your favorite non-triangular invertible 2 by 2 matrix to find a counterexample).

b) (10 points) True. It is certainly the case that $E0 = 0$.

4. a) (10 points) Permutation.

b) (18 points) Well, $P\mathbf{e} = \mathbf{e}$ for any permutation matrix, because each row has eight zero's and one one. So by part a), $A\mathbf{e} = \mathbf{e} + 2\mathbf{e} + \ldots + 9\mathbf{e} = 45\mathbf{e}$. But since the transpose of a permutation matrix is also a permutation matrix, $A^t\mathbf{e} = 45\mathbf{e}$ also. But $\mathit{rank}\begin{pmatrix} \mathbf{e} & 45\mathbf{e} & 45\mathbf{e} \end{pmatrix} = 1$.