18.06 Professor Edelman Quiz 1 September 29, 2008

Your PRINTED name is: _____

Please circle your recitation:

Grading	
1	
2	
3	
4	
	_____

1)	T 10	2-131	J.Yu	2-348	4-2597	`jyu`
2)	T 10	2-132	J. Aristoff	2-492	3-4093	`jeffa`
3)	T 10	2-255	Su Ho Oh	2-333	3-7826	`suho`
4)	T 11	2-131	J. Yu	2-348	4-2597	`jyu`
5)	T 11	2-132	J. Pascaleff	2-492	3-4093	`jpascale`
6)	T 12	2-132	J. Pascaleff	2-492	3-4093	`jpascale`
7)	T 12	2-131	K. Jung	2-331	3-5029	`kmjung`
8)	T 1	2-131	K. Jung	2-331	3-5029	`kmjung`
9)	T 1	2-136	V. Sohinger	2-310	4-1231	`vedran`
10)	T 1	2-147	M Frankland	2-090	3-6293	`franklan`
11)	T 2	2-131	J. French	2-489	3-4086	`jfrench`
12)	T 2	2-147	M. Frankland	2-090	3-6293	`franklan`
13)	T 2	4-159	C. Dodd	2-492	3-4093	`cdodd`
14)	T 3	2-131	J. French	2-489	3-4086	`jfrench`
15)	T 3	4-159	C. Dodd	2-492	3-4093	`cdodd`

1 (18 pts.) Consider the equation $Ax = b$:

$$\begin{bmatrix} 1 & 0 \\ 4 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

(a) Put the equation into echelon form $Rx = d$.

(b) For which b are there solutions?

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2 (24 pts.) The matrix A has two special solutions:

$$x_1 = \begin{bmatrix} c \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad x_2 = \begin{bmatrix} d \\ 0 \\ 1 \end{bmatrix}.$$

(a) Describe all the possibilities for the number of columns of A.

(b) Describe all the possibilities for the number of rows of A.

(c) Describe all the possibilities for the rank of A.

Briefly explain your answers.

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3 (30 pts.) Let A be any matrix and R its row reduced echelon form. Answer True or False to the statements below and briefly explain. (Note, if there are any counterexamples to a statement below you must choose false for that statement.)

(a) If x is a solution to $Ax = b$ then x must be a solution to $Rx = b$.

(b) If x is a solution to $Ax = 0$ then x must be a solution to $Rx = 0$.

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4 (28 pts.) A Sudoko puzzle solution such as the example on the last page is a $9x9$ matrix A that among other properties has the numbers 1 through 9 once in every row and in every column.

Hint 1: There is no need to compute at all to solve this problem, and familiarty with Sudoko puzzles are unlikely to help or hurt.

Hint 2: 1+2+3+...+9=45.

(a) All such matrices A can be written as

$$A = P_1 + 2P_2 + 3P_3 + \ldots + 8P_8 + 9P_9,$$

where the matrices $P_1, \ldots, P_9$ are what kind of matrices? (Looking for what we consider the best possible one word answer. Square would be correct, but would not be acceptable.)

(b) Let e be the 9×1 vector of nine 1's. What is the rank of the 9x3 matrix whose columns are e, Ae, and $A^T e$ for any such matrix A. Explain your answer.

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