SOLUTIONS TO PROBLEM SET 2

Problem 1. 1) (2.5 points) $AB = A\begin{pmatrix} B_1 & B_2 & B_3 & B_4 \end{pmatrix} = \begin{pmatrix} AB_1 & AB_2 & AB_3 & AB_4 \end{pmatrix}$.

2) (2.5 points) $AB = \begin{pmatrix} A^1 \\ A^2 \end{pmatrix} B = \begin{pmatrix} A^1B \\ A^2B \end{pmatrix}$.

3) (2.5 points) $AB = \begin{pmatrix} A^1 \\ A^2 \end{pmatrix} \begin{pmatrix} B_1 & B_2 & B_3 & B_4 \end{pmatrix} = \begin{pmatrix} A^1B_1 & A^1B_2 & A^1B_3 & A^1B_4 \\ A^2B_1 & A^2B_2 & A^2B_3 & A^2B_4 \end{pmatrix}$.

4) (2.5 points) $AB = \begin{pmatrix} A_1 & A_2 & A_3 \end{pmatrix} \begin{pmatrix} B^1 \\ B^2 \\ B^3 \end{pmatrix} = \begin{pmatrix} A_1B^1 + A_2B^2 + A_3B^3 \end{pmatrix}$.

Problem 2. Well, the imaginary part is $i(B\mathbf{x}+A\mathbf{y})$, so the final matrix must be $\begin{pmatrix} A & -B \\ B & A \end{pmatrix}$.

Problem 3. When we put $c = 0$, the second row is all zeros, so the matrix is noninvertible. When $c = 7$, the second and third column are equal, and when $c = 2$, the first and second row are equal.

Problem 4. You are given A^{-1}, A and D, so checking $A^{-1} = DAD$ is a routine matrix multiplication. This equation implies $ADAD = I$, or equivalently $(AD)^2 = I$, thus AD is its own inverse.

Problem 5. The first equation is $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, so we see that $c_1 = 4$, and $c_1 + c_2 = 5$, so $c_2 = 1$, and finally $c_1 + c_2 + c_3 = 6$, so $c_3 = 1$ as well. Then we solve $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$, to get that $x_3 = 1$, $x_2 = 0$, and $x_1 = 3$. $A = LU = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

Problem 6. The fact that P_1P_2 is a permuatation matrix follows from the fact that P_1M permutes the rows of M, for any matrix M. If you permute the rows of a permutation matrix, you get another (just by definition). For the first example, take $P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. For the second example, let P_3 be the identity matrix.

Problem 7. a) (2.5 points) $(A^2 - B^2)^t = (A^2)^t - (B^2)^t = (A^t)^2 - (B^t)^2 = A^2 - B^2$.

b) (2.5 points) $(A+B)(A-B) = A^2 - B^2 + BA - AB$. The first term is symmetric by a). On the other hand, $(BA - AB)^t = A^tB^t - B^tA^t = AB - BA$. So this will fail for any two noncommuting matrices.

c) (2.5 points) $(ABA)^t = A^tB^tA^t = ABA$.

d) (2.5 points) $(ABAB)^t = B^tA^tB^tA^t = BABA$, so, noncommuting matrices will make this fail.

Problem 8. a) (5 points) $A^t\mathbf{y} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} y_{BC} \\ y_{CS} \\ y_{BS} \end{pmatrix} = \begin{pmatrix} y_{BC}+y_{BS} \\ y_{CS}-y_{BC} \\ -y_{CS}-y_{BS} \end{pmatrix} = \begin{pmatrix} x_B-x_C+x_B-x_S \\ x_C-x_S-x_B+x_C \\ x_S-x_C-x_B+x_S \end{pmatrix} = \begin{pmatrix} 2x_B-x_C-x_S \\ 2x_C-x_S-x_B \\ 2x_S-x_C-x_B \end{pmatrix}$.

b) (5 points) left to the reader.