PRACTICE PROBLEMS FOR EXAM 3

Not to be turned in.

Problem 1. Let U be a unitary matrix and Λ an eigenvalue matrix for a symmetric matrix. What kind of matrix is $A = U^H \Lambda U$? Must the eigenvalues of A be real? Must A be positive definite (positive eigenvalues)? Must the eigenvectors be real? Must the diagonal elements be real? Must the off-diagonal elements be real?

Problem 2. Let $\mathbf{x}$ be a complex eigenvector of a complex matrix of unit length. For which constants c is $c\mathbf{x}$ not an eigenvector? What constants c preserve the property that $c\mathbf{x}$ is an eigenvector of unit length?

Problem 3. If A is an $n \times n$ Markov matrix, then prove that $0 \leq Tr(A) \leq n$. Prove that a 2×2 Markov matrix has $-1 \leq det(A) \leq 1$.

Problem 4. Prove that a 3×3 nonidentity Markov matrix can not have a triple eigenvalue at 1.

Problem 5. If possible find an invertible M such that $M \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} M^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{pmatrix}$.If not why can one not exist?

Problem 6. A is $n \times n$ diagonalizable and has eigenvalues 0 and 1. What is A^2?

Problem 7. If A is 2×2 and has eigenvalues -1 and 1 must A^4 be diagonalizable? What if A were 3×3?