18.06 Professor Edelman Final Exam December 18, 2008

Grading

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Your PRINTED name is: ____________________

Please circle your recitation:

1)	T 10	2-131	J.Yu	2-348	4-2597	jyu
2)	T 10	2-132	J. Aristoff	2-492	3-4093	jeffa
3)	T 10	2-255	Su Ho Oh	2-333	3-7826	suho
4)	T 11	2-131	J. Yu	2-348	4-2597	jyu
5)	T 11	2-132	J. Pascaleff	2-492	3-4093	jpascale
6)	T 12	2-132	J. Pascaleff	2-492	3-4093	jpascale
7)	T 12	2-131	K. Jung	2-331	3-5029	kmjung
8)	T 1	2-131	K. Jung	2-331	3-5029	kmjung
9)	T 1	2-136	V. Sohinger	2-310	4-1231	vedran
10)	T 1	2-147	M Frankland	2-090	3-6293	franklan
11)	T 2	2-131	J. French	2-489	3-4086	jfrench
12)	T 2	2-147	M. Frankland	2-090	3-6293	franklan
13)	T 2	4-159	C. Dodd	2-492	3-4093	cdodd
14)	T 3	2-131	J. French	2-489	3-4086	jfrench
15)	T 3	4-159	C. Dodd	2-492	3-4093	cdodd

1 (9 pts.) Given real numbers a, b and c, find x, y and z such that the matrix B below is guaranteed to be singular with real eigenvalues and orthogonal eigenvectors.

$$B = \begin{bmatrix} a & b & a+b \\ b & c & b+c \\ x & y & z \end{bmatrix}.$$

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2 (12 pts.) The matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & p \end{bmatrix}.$$

(a) What are the eigenvalues of A (possibly in terms of p)?

(b) If p is not 0, find an eigenvector that is not in the nullspace.

(c) What are the singular values of A (possibly in terms of p)?

(d) Find a nonzero solution $u(t)$ to $du/dt = (A + 2009I)u$. Check that your answer is correct. (Note that $A + 2009I$ is the matrix above with the upcoming new year added to the diagonal elements.)

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3 (8 pts.) A 4x4 square matrix A has singular values $3, 2, 1$, and 0. Find an eigenvalue of A. Briefly explain your answer.

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4 (9 pts.) The square matrix A has QR decomposition $A = QR$ where Q is orthogonal and R is upper triangular with diagonal elements all equal to 1.

(a) What is the determinant of A^TA?

(b) What are all the pivots of A^TA?

(c) Are the matrices QR and RQ similar?

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5 (5 pts.) All matrices in this question are $n \times n$. We have that $C = A^{-1}BX$. Propose an X which guarantees that B and C have the same eigenvalues.

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6 (12 pts.) All you are told about a 3×3 matrix A is that five of the nine entries are 1, and the other four are 0. For the ranks below, exhibit a matrix A with this property, or else briefly (but convincingly) argue that it is impossible.

(a) A has rank 0.

(b) A has rank 2.

(c) A has rank 3.

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7 (10 pts.) (Do **two** of the three problems below. Please avoid any confusion to the graders as to which two you chose.)

(a) All you are told about a 100 by 100 matrix is that all of its entries are **even** integers. Must the determinant be odd? Must the determinant be even? Argue your answer convincingly.

(b) Give an example, if possible, of a 100 by 100 matrix with odd integer entries but an even determinant.

(c) All you are told about a 100 by 100 matrix is that all of its entries are **odd** integers. Must the determinant be odd? Must the determinant be even? Argue your answer convincingly.

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8 (15 pts.) The functions of the form

$$f(x) = c_1 + c_2 e^x + c_3 e^{2x},$$

form a three dimensional vector space V.

(a) The transformation d/dx can be written as a 2x3 matrix when the domain is specified to have basis $\{1, e^x, e^{2x}\}$, and the range has basis $\{e^x, e^{2x}\}$. Write down this 2x3 matrix.

(b) On the above three dimensional vector space V, is the evaluation of f at $x = 7$ a linear transformation from that space to R?

(c) On the above three dimensional vector space V, is the transformation that takes $f(x)$ to $\int_0^x f(t)dt$ a linear transformation from that three dimensional space to itself (from V to V) ?

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9 (20 pts.) Suppose an n by n matrix has the property that its nullspace is equal to its column space.

(a) Can the matrix be the zero matrix?

(b) Possibly in terms of n, what is the rank of the matrix?

(c) What are the eigenvalues of this matrix? (Briefly explain your answer. Hint: It might be useful to consider applying A more than once in some way.)

(d) Give an example of a 2 by 2 such matrix.

(e) Perhaps using the previous case twice somehow, give an example of a 4 x 4 such matrix.

Have a great holiday vacation! Thank you for taking linear algebra.