18.06 Problem Set 4

Due Wednesday, 10 October 2007 at 4 pm in 2-106.

Problem 1: Decide whether the following set of vectors are linearly dependent or independent. (Give reasons)
(a) $(1,2,3),(2,3,1),(3,1,2)$.
(b) $(1,1,0,0),(1,-1,0,0),(1,0,0,0)$.
(c) $(0,0,0),(1,4,5),(1,0,4)$.
(d) $1, 1+x, 1+x^2$ (in the vector space of polynomials).
(e) $\mathbf{v}_1-\mathbf{v}_2, \mathbf{v}_2-\mathbf{v}_3, \mathbf{v}_3-\mathbf{v}_4, \mathbf{v}_4-\mathbf{v}_1$.

Problem 2: Find a basis of the following vector spaces.
(a) All vectors in $\mathbb{R}^3$ whose components are equal.
(b) All vectors in $\mathbb{R}^4$ whose components add to zero and whose first two components add to equal twice the fourth component.
(c) All vectors in $\mathbb{R}^4$ that are perpendicular to $(1,0,1,0)$.
(d) All anti-symmetric 3×3 matrices.
(e) All polynomials $p(x)$ whose degree is no more than 3 and satisfies $p(0)=0$.

Problem 3: Do problem 13 from section 3.5 (P 169) in your book.

Problem 4: Do problem 2 from section 3.6 (P 180) in your book.

Problem 5: Do problem 22 from section 3.6 (P 182) in your book.

Problem 6: Do problem 23 from section 3.6 (P 182) in your book.

Problem 7: True or false: (Give reasons)
(a) If the row space of A equals the column space of A, then $A^T=A$.
(b) If the column vectors of a matrix are dependent, so are the row vectors.
(c) The matrices A and $-A$ have the same four subspaces.
(d) The rows of a matrix are a basis of the row space.
(e) The column space of a 3×4 matrix has the same dimension as its row space.

Problem 8: Give the matrix $A = \begin{pmatrix} 1 & 1 & 2 & 4 \\ 3 & c & 2 & 8 \\ 0 & 0 & 2 & 2 \end{pmatrix}$ depending on c.

(a) find a basis for the column space of A.
(b) find a basis for the nullspace of A.
(c) find the complete solution x to $A\mathbf{x} = \begin{pmatrix} 1 \\ c \\ 0 \end{pmatrix}$.

Problem 9: Do problem 8 from section 8.2 (P 421) in your book. (Note that the graph is the second one on page 420.)

Problem 10: (a) Use MATLAB to construct a random 10×5 matrix A and a random 5×9 matrix B. Then use MATLAB to find bases for the four subspaces of the matrix AB. (Hints: The commands $A = rand(10,5); B = rand(5,9)$; will give you the two random matrix, and the command $[R,p] = rref(A)$ returns the row-reduced echelon form R of A and a list p of the pivot columns.)
(b) The random matrices you constructed almost certainly have either full row or full column rank. Why?
(c) Suppose you were inventing homework problems for your classmates, and wanted to come up with random 7×8 matrix that is only rank 5...but it shouldn't be "obviously" rank 5 (i.e. it should be hard to tell just by looking at the matrix that the rows and columns are linearly dependent). Figure out a way to do this in Matlab, come up with a random 7×8 rank-5 matrix, check that it is rank 5, and with the help of the rref command find a basis for its four fundamental subspaces. (Hint: use a combination of random rank-5 matrices.)