18.06 Professor Strang Quiz 2 November 2, 2005

Your PRINTED name is: _____

Grading

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Please circle your recitation:

1)	M 2	2-131	P. Lee	2-087	2-1193	`lee`
2)	M 2	2-132	T. Lawson	4-182	8-6895	`tlawson`
4)	T 10	2-132	P-O. Persson	2-363A	3-4989	`persson`
5)	T 11	2-131	P-O. Persson	2-363A	3-4989	`persson`
6)	T 11	2-132	P. Pylyavskyy	2-333	3-7826	`pasha`
7)	T 12	2-132	T. Lawson	4-182	8-6895	`tlawson`
8)	T 12	2-131	P. Pylyavskyy	2-333	3-7826	`pasha`
9)	T 1	2-132	A. Chan	2-588	3-4110	`alicec`
10)	T 1	2-131	D. Chebikin	2-333	3-7826	`chebikin`
11)	T 2	2-132	A. Chan	2-588	3-4110	`alicec`
12)	T 3	2-132	T. Lawson	4-182	8-6895	`tlawson`

1 (30 pts.) The matrix A has a varying $1 - x$ in the $(1, 2)$ position:

$$A = \begin{bmatrix} 2 & \mathbf{1 - x} & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 4 \\ 1 & 1 & 3 & 9 \end{bmatrix}$$

(a) When $x = 1$ compute $\det A$. What is the $(1, 1)$ entry in the inverse when $x = 1$?

(b) When $x = 0$ compute $\det A$.

(c) How do the properties of the determinant say that $\det A$ is a linear function of x? For any x compute $\det A$. For which x's is the matrix singular?

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2 (30 pts.) This matrix Q has orthonormal columns q_1, q_2, q_3:

$$Q = \begin{bmatrix} .1 & .5 & a \\ .7 & .5 & b \\ .1 & -.5 & c \\ .7 & -.5 & d \end{bmatrix}.$$

(a) What equations must be satisfied by the numbers a, b, c, d? Is there a unique choice for those numbers, apart from multiplying them all by -1?

(b) Why is $P = QQ^{\mathrm{T}}$ a projection matrix? (Check the two properties of projections.) Why is QQ^{T} a singular matrix? Find the determinants of $Q^{\mathrm{T}}Q$ and QQ^{T}.

(c) Suppose Gram-Schmidt starts with those same first two columns and with the third column $a_3 = (1, 1, 1, 1)$. What third column would it choose for q_3? You may leave a square root not completed (if you want to).

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3 (40 pts.) Our measurements at times $t = 1, 2, 3$ are $b = 1, 4$, and b_3. We want to fit those points by the nearest line $C + Dt$, using least squares.

(a) Which value for b_3 will put the three measurements on a straight line? *Which line is it*? Will least squares choose that line if the third measurement is $b_3 = 9$? (Yes or no).

(b) What is the linear system $Ax = b$ that would be solved exactly for $x = (C, D)$ if the three points do lie on a line? Compute the projection matrix P onto the column space of A. Remember the inverse

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

(c) What is the rank of that projection matrix P? How is the column space of P related to the column space of A? (You can answer with or without the entries of P computed in (b).)

(d) Suppose $b_3 = 1$. Write down the equation for the best least squares solution $\widehat{x}$, and show that the best straight line is horizontal.

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