18.06, Fall 2004, Problem Set 4

Due before 4PM on Wednesday October 6th, 2004, in the boxes in 2-106. No late homework will be accepted. Don't forget to write your name, recitation section **and the names of students you have collaborated with** on the problem set. There is one box for each recitation section. For full credit, please be sure to show and explain your work. Exercises refer to the 3rd edition of the textbook.

Reading assignment: Rest of chapter 3.

1. Consider
$$A = \begin{bmatrix} 0 & 0 & 2 & -2 & 1 & 2 \\ 3 & 6 & 0 & 9 & 0 & 3 \\ 1 & 2 & 0 & 3 & 1 & 3 \\ -1 & -2 & 2 & -5 & 0 & -1 \end{bmatrix}.$$
   (a) Compute the reduced row echelon form R for A.
   (b) What is the rank of A?
   (c) Give a vector b (if any) such that $Ax = b$ has no solution.
   (d) Give all solutions to $Ax = \begin{bmatrix} 22 \\ 24 \\ 16 \\ 6 \end{bmatrix}$.
   (e) Is there a vector b with exactly one solution to $Ax = b$?
   (f) Compute $A^T A$.
   (g) What is the rank of $A^T A$?

2. Consider the space F spanned by the 4 vectors $v_1 = (4, 2, 4, 2)$, $v_2 = (-1, 4, 5, 10)$, $v_3 = (-5, 2, 1, 8)$ and $v_4 = (6, 6, 10, 10)$.
   (a) Are the v_i's linearly independent?
   (b) Give a basis of F.
   (c) What is the dimension of F?
   (d) Are $v_1 + 2v_2 + 3v_3$, $v_1 - v_2$ and v_4 linearly independent?

3. Consider the subspace F of all 3×3 *symmetric* matrices with zeroes on the diagonal.
   (a) Give a basis of F. Justify.
   (b) More generally, what is the dimension of the subspace of symmetric $n \times n$ matrices with zeroes on the diagonal?

4. Let A be an $m \times n$ matrix of rank r. Let $v_1, v_2, \cdots, v_n$ denote the columns of A. Let $k < r$ and assume that the first k vectors $v_1, v_2, \cdots, v_k$ are linearly independent. Prove (by contradiction) that you can always find an index l (with $l > k$ and $l \leq n$) such that $v_1, v_2, \cdots, v_{k-1}, v_k, v_l$ (take the first k columns and also the lth one) are linearly independent.

5. Exercise 14 of section 3.6 on page 181.