Your name is:

Please circle your recitation:

1)	M2	2-131	Holm	2-181	3-3665	tsh@math
2)	M2	2-132	Dumitriu	2-333	3-7826	dumitriu@math
3)	M3	2-131	Holm	2-181	3-3665	tsh@math
4)	T10	2-132	Ardila	2-333	3-7826	fardila@math
5)	T10	2-131	Czyz	2-342	3-7578	czyz@math
6)	T11	2-131	Bauer	2-229	3-1589	bauer@math
7)	T11	2-132	Ardila	2-333	3-7826	fardila@math
8)	T12	2-132	Czyz	2-342	3-7578	czyz@math
9)	T12	2-131	Bauer	2-229	3-1589	bauer@math
10)	T1	2-132	${\bf Ingerman}$	2-372	3-4344	ingerman@math
11)	T1	2-131	Nave	2-251	3-4097	nave@math
12)	T2	2-132	${\bf Ingerman}$	2-372	3-4344	ingerman@math
13)	T2	1-150	Nave	2-251	3-4097	nave@math

1 (30 pts.) Suppose the matrix A has reduced row echelon form R:

$$A = \left[egin{array}{cccc} 1 & 2 & 1 & b \ 2 & a & 1 & 8 \ & ({
m row} & 3) \end{array}
ight], \quad R = \left[egin{array}{cccc} 1 & 2 & 0 & 3 \ 0 & 0 & 1 & 2 \ 0 & 0 & 0 & 0 \end{array}
ight]$$

- (a) What can you say immediately about row 3 of A?
- (b) What are the numbers a and b?
- (c) Describe all solutions of Rx = 0. Circle the spaces that are the same for A as for R: (row space)(column space)(null space).

2 (30 pts.) (a) Find the number c that makes this matrix singular (not invertible):

$$A = \left[egin{array}{ccc} 1 & 2 & 3 \ 1 & 5 & 6 \ 2 & 6 & c \end{array}
ight]$$

- (b) If c=20 what are the column space C(A) and the nullspace N(A)?

 Describe them in this specific case (not just repeat their definitions).

 Also describe $C(A^{-1})$ and $N(A^{-1})$ for the inverse matrix!
- (c) With c=20 factor the matrix into A=LU (lower triangular L and upper triangular U).

- **3** (40 pts.) Suppose A is an m by n matrix of rank r.
 - (a) If Ax = b has a solution for every right side b, what is the column space of A?
 - (b) In part (a), what are all equations or inequalities that must hold between the numbers m, n, and r.
 - (c) Give a specific example of a 3 by 2 matrix A of rank 1 with first row [2 5]. Describe the column space C(A) and the nullspace N(A) completely.
 - (d) Suppose the right side b is the same as the first column in your example (part c). Find the complete solution to Ax = b.