14.272 Problem Set 3
Cost functions and Ramsey Prices

Andrew Sweeting

April 7, 2002

Question 1: Cost Function Properties

The point of this question is to work through the definitions you saw in class. The main point to take away is which cost features make naturally monopoly likely and which features make it sensible to bundle products in the same firm.

Cost function is

$$C(q_1, q_2) = q_1 + q_2 - (q_1.q_2)^{\frac{1}{3}}$$

(a) There are **no** economies of scale which is most easily seen by noticing that

$$C(kq_1, kq_2) > kC(q_1, q_2) fork > 1, q_1, q_2 > 0$$

as

$$C(kq_1, kq_2) = kq_1 + kq_2 - k^{\frac{2}{3}}.(q_1.q_2)^{\frac{1}{3}}$$

and

$$k^{\frac{2}{3}} < k$$

You can also get the same result by calculating

$$S(q) = \frac{C(q)}{\sum q_i.\frac{\partial C}{\partial q_i}} = \frac{q_1 + q_2 - (q_1.q_2)^{\frac{1}{3}}}{q_1 + q_2 - \frac{2}{3}(q_1.q_2)^{\frac{1}{3}}} < 1$$

Increasing ray average cost is implied by diseconomies of scale

$$AC(kq_1, kq_2) = \frac{C(kq)}{k(q_1 + q_2)}$$

$$\frac{\partial AC(kq_1, kq_2)}{\partial k} = \frac{1}{3}.k^{-\frac{4}{3}}.\frac{(q_1.q_2)^{\frac{1}{3}}}{(q_1 + q_2)}$$

which we evaluate a $k = 1$

$$= \frac{1}{3}.\frac{(q_1.q_2)^{\frac{1}{3}}}{(q_1+q_2)} > 0$$

(b) Economies of scope are present

$$C(q_1, 0) + C(0, q_2) < C(q_1, q_2) for q_1, q_2 > 0$$

You can also get transray convexity by showing that $C_{11} > 0, C_{22} > 0$ and $C_{12} < 0$ which are sufficient conditions in the two product firm case.

(c) Cost complementarity is defined by

$$\frac{\partial^2 C}{\partial q_i \partial q_j} \leq 0 \ \forall i, j$$

In this case

$$\frac{\partial^2 C}{\partial q_1 \partial q_2} = -\frac{1}{9}.q_1^{-\frac{2}{3}}.q_2^{-\frac{2}{3}} < 0$$

but

$$\begin{aligned} \frac{\partial^2 C}{\partial q_1^2} &= \frac{2}{9} q_1^{-\frac{5}{3}} q_2^{\frac{1}{3}} > 0 \\ \frac{\partial^2 C}{\partial q_2^2} &= \frac{2}{9} q_2^{-\frac{5}{3}} q_1^{\frac{1}{3}} > 0 \end{aligned}$$

In this case the lack of economies of scale mean that despite the presence of an economy of scope there is not a cost complementarity.

(d) The cost function is not subadditive. You can use the fact that

$$C(kq_1, kq_2) > kC(q_1, q_2) for k > 1$$

as discussed above. This means that for any output vector with positive amount of both products we can produce it more cheaply by breaking production up into two firms. Hence the cost function cannot be subadditive.

Question 2: Ramsey Pricing

(a) It should be intuitive that the solution here is

$$p_n = 10, p_d = 11$$

as for these prices

$$q_n = 82, q_d = 89$$

so the network is not fully used at night even when the calls are priced at marginal cost. Hence following the peak load pricing argument all the capacity costs should be borne by daytime calls (which create the marginal need for capacity). Note also in this case that there is no need to worry about breakeven constraints because there is not decreasing costs with volume of calls.

If you want to set up the problem formally then

$$\max_{q_n, q_d} W = \int_0^{q_d} (100 - q).dq + \int_0^{q_n} (92 - q).dq - 10(q_d + q_n) - q_d$$

where we make the assumption (which can be verified at the end) that the optimal $q_d > q_n$.

The FOCs are

$$\begin{aligned} \frac{\partial W}{\partial q_d} &= 100 - q_d - 11 = 0 so q_d = 89 \\ \frac{\partial W}{\partial q_n} &= 100 - q_n - 11 = 0 so q_n = 82 \end{aligned}$$

which obviously have the same prices as above.

(b) I assume that the benchmark for uniform prices is $p = 6$ although there would certainly be other uniform prices which would do better.

Welfare under uniform prices of 6 is

$$\begin{aligned} & \int_0^{94} (100 - q).dq + \int_0^{86} (92 - q).dq - 10(100 - 6 + 92 - 6) - 1(100 - 6) \\ = & \quad 4982 + 4214 - 1800 - 94 \\ = & \quad 7302 \end{aligned}$$

Welfare under Ramsey prices is

$$\begin{aligned} & \int_0^{89} (100 - q).dq + \int_0^{82} (92 - q).dq - 10(100 - 11 + 92 - 10) - 1(100 - 11) \\ = & \quad 4939.5 + 4182 - 1710 - 89 \\ = & \quad 7322.5 \end{aligned}$$

Therefore as long as the fixed cost is less than 20.5 we should switch to Ramsey prices.

(c) Subsidy-freeness for a two product firm is defined by

$$\begin{aligned} C(q_1, q_2) &= p_1.q_1 + p_2.q_2 \\ C(q_1, 0) &\geq p_1.q_1 \\ C(0, q_2) &\geq p_2.q_2 \end{aligned}$$

These conditions are satisfied here as Ramsey prices will make the firm breakeven and

$$\begin{aligned} C(q_d, 0) &= 979 = p_d.q_d \\ C(0, q_n) &= 902 > 820 = p_n.q_n \end{aligned}$$

Question 3

(a) Easy : break even with a uniform price is given by

$$\begin{aligned} p(30 - 0.5p) + p(120 - 2p) - 1800 &= 0 \\ 150p - 2.5p^2 - 1800 &= 0 \end{aligned}$$

and you can then either use the quadratic formula to get p=16.58 as a solution or simply plug is 16.58 and show it is a solution.

(b) Linear Ramsey prices are the same as in (a). Why? The demand curve of B is simply four times the demand of A - imagine for example, that A is a household of 1 person and B is a household of 4 people with all individuals identical. It should be intuitive that each identical consumer should have the same price. You can show the same thing algebraically by noting that demands are independent and that $\varepsilon_A = \varepsilon_B$ which implies Ramsey prices are the same. As the firm must breakeven (a) will then give the solution.

(c) We want to find an arrangement that makes one party better off and the two other parties no worse off. Imagine the following

Tariff 1: p=16.58, no fixed fee
Tariff 2: p=0, fixed fee of 1440.

This is exactly the kind of arrangement which Nancy considered in class to show that you can do better than linear Ramsey prices. (Aside for PFers: the logic for why this is the natural deviation to consider is similar to the Mirrlees tax result that you can always do better by charging a zero marginal tax rate on the last dollar of the highest earning individual).

A chooses tariff 1 and is as well off as in (a) and the firm gets the same revenue from A. B chooses tariff 2 paying the same revenue as in (a), ensuring that the firm is as well off as in (a), but B is better off as he consumes more while paying as much as before.

(d) To maximize consumer surplus you set price equal to marginal cost and recover all the costs via a fixed fee. In this case you should offer a single tariff of marginal price equals zero and a fixed fee of 900. A gets no surplus, while B gets surplus of 2,700. The firm gets revenue of 1,800 and covers costs. Obviously an equity concern might lead one to prefer (a) as consumer A now gets no surplus.

(e) If the firm can monitor quantity consumed by individuals then it could offer a zero "marginal price" but a fixed fee of 900 to consume up to 30 units and a fixed fee of 1100 to consume more than 30 units. A will consume 30 and pay the fixed fee of 900 and be left with no surplus. B will consume 120 and will pay a fixed fee of 1,100. It easy to check that B will prefer to do this than limit its consumption (surplus net of fee is 675 if only consumes 30, otherwise it is 2500).