

Lecture Notes on Fluid Dynamics

(1.63J/2.21J)

by Chiang C. Mei, 2004

7-9-2upwell.tex

7.9 Coastal upwelling in a two-layered sea

When a steady wind blows along the shore, an Ekman drift is induced that leads to mass flux perpendicular to the wind. Consider the eastern coast in the northern hemisphere. If the wind blows to the north, so that the coast is on its left (west), there is an Ekman drift with mass moving away from the coast to the east. Fluid must be replenished from below, so that the interface must rise, see figure 7.9.1. This is called *coastal upwelling*, first analyzed theoretically by Kozo Yoshida (1959). This phenomenon is important to life in the ocean. Small organisms such as phytoplanktons need nutrient and sun light to prosper. If upwelling occurs near a coastal water, nutrients can be transported from great depth to near the sea surface where sun light is rich. Fishes are therefore more bountiful.

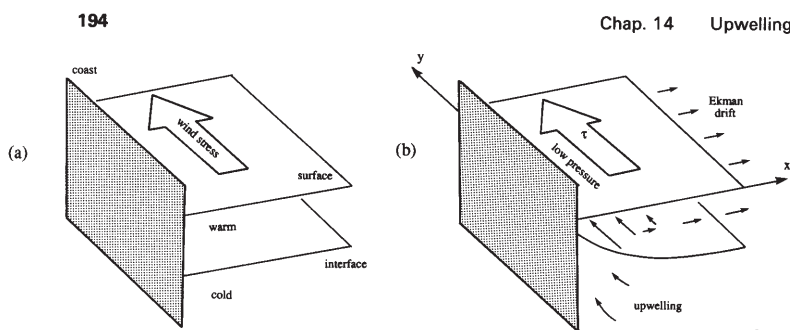


Figure 14-1 Schematic development of coastal upwelling.

Figure 7.9.1: Physical mechanism of coastal upwelling. From Cushman-Roisin

We consider a spatially uniform wind blowing along a coastline $x = 0$. According to the normal mode formulation the equations for modes $k = 1, 2$ are

$$\frac{\partial \bar{U}_k}{\partial t} - f \bar{V}_k = -g \beta_k h \frac{\partial \bar{\zeta}_k}{\partial x} \quad (7.9.1)$$

$$\frac{\partial \bar{V}_k}{\partial t} + f \bar{U}_k = \frac{\tau_k}{\rho} \quad (7.9.2)$$

$$\frac{\partial \bar{\zeta}_k}{\partial t} + \frac{\partial \bar{U}_k}{\partial x} = 0 \quad (7.9.3)$$

where \bar{U}_k, \bar{V}_k and $\bar{\zeta}_k$ are related to the depth-integrated fluxes U, V, ζ in the upper layer and U', V', ζ' in the lower layer by

$$\bar{U}_k = a_k U + b_k U', \quad \bar{V}_k = a_k V + b_k V', \quad \bar{\zeta}_k = a_k \zeta + (b_k - a_k) \zeta' \quad (7.9.4)$$

and the normal form of wind forcing is

$$\tau_k = a_k \tau_y^S \quad (7.9.5)$$

Assume that the wind oscillates in time at the frequency ω so that

$$\tau_y^S = \Re \left(i \tau_o e^{-i\omega t} \right) = \tau_o \sin \omega t \quad (7.9.6)$$

Let us look for the response that is also sinusoidal in time, and write the solutions as

$$(\bar{U}_k, \bar{V}_k, \bar{\zeta}_k) = \Re \left\{ (U_k, V_k, \zeta_k) e^{-i\omega t} \right\}$$

then

$$-i\omega U_k - f V_k = -g\beta_k h \frac{\partial \zeta_k}{\partial x} \quad (7.9.7)$$

$$f U_k - i\omega V_k = \frac{\tau_k}{\rho} \quad (7.9.8)$$

$$i\omega \zeta_k + \frac{\partial U_k}{\partial x} = 0 \quad (7.9.9)$$

From Eqns. (7.9.7) and (7.9.8) we solve for U_k and V_k ,

$$U_k = \frac{\begin{vmatrix} -g\beta_k h \frac{\partial \bar{\zeta}_k}{\partial x} & -f \\ \frac{\tau_k}{\rho} & -i\omega \end{vmatrix}}{\begin{vmatrix} -i\omega & -f \\ f & -i\omega \end{vmatrix}} = \frac{i\omega g\beta_k h \frac{\partial \zeta_k}{\partial x} + f \frac{\tau_k}{\rho}}{f^2 - \omega^2} \quad (7.9.10)$$

and

$$V_k = \frac{\begin{vmatrix} -i\omega & -g\beta_k h \frac{\partial \bar{\zeta}_k}{\partial x} \\ f & \frac{\tau_k}{\rho} \end{vmatrix}}{\begin{vmatrix} -i\omega & -f \\ f & -i\omega \end{vmatrix}} = \frac{f g\beta_k h \frac{\partial \zeta_k}{\partial x} - i\omega \frac{\tau_k}{\rho}}{f^2 - \omega^2} \quad (7.9.11)$$

Substituting Eqn. (7.9.10) into (7.9.9), we get

$$-i\omega \zeta_k + \frac{1}{f^2 - \omega^2} \left(i\omega g\beta_k h \frac{\partial^2 \zeta_k}{\partial x^2} \right) = 0$$

or,

$$\frac{\partial^2 \zeta_k}{\partial x^2} - \left(\frac{f^2 - \omega^2}{g\beta_k h} \right) \zeta_k = 0. \quad (7.9.12)$$

Let us limit our attention to low frequencies so that so that $f^2 > \omega^2$. The solution bounded at $x \sim \infty$ is,

$$\zeta_k = A_k e^{-x/R_k}, \quad (7.9.13)$$

where

$$R_k = \frac{\sqrt{g\beta_k h}}{\sqrt{f^2 - \omega^2}}. \quad (7.9.14)$$

is the modified Rossby radius of deformation. As *b.c* at $x = 0$, we require $U_k = 0$, hence from (7.9.10)

$$\begin{aligned} i\omega g\beta_k h \frac{\partial \zeta_k}{\partial x} &= -\frac{f}{\rho\tau_k} \\ i\omega g\beta_k h A_k \left(-\frac{1}{R_k}\right) &= -\frac{f\tau_k}{\rho} \end{aligned}$$

The coefficient is

$$A_k = \frac{\frac{f\tau_k}{\rho} R_k}{i\omega g\beta_k h} \quad (7.9.15)$$

therefore

$$\zeta_k = \frac{\frac{f\tau_k}{\rho} R_k}{i\omega g\beta_k h} e^{-x/R_k}. \quad (7.9.16)$$

Recall for Mode 1 (barotropic or surface mode):

$$\beta_1 = \frac{h+h'}{h}, \quad a_1 = 1, \quad \bar{\tau}_1 = \tau_1 e^{-i\omega t} = \tau_y^S = i\tau_o e^{-i\omega t} \quad (7.9.17)$$

then

$$\tau_1 = i\tau_o, \quad R_1 = \frac{\sqrt{g\frac{h+h'}{h}h}}{\sqrt{f^2 - \omega^2}} = \frac{\sqrt{g(h+h')}}{\sqrt{f^2 - \omega^2}} \quad (7.9.18)$$

hence we have

$$\zeta_1 = \frac{\frac{f\tau_o}{\rho} R_1}{\omega g(h+h')} e^{-x/R_1} \quad (7.9.19)$$

For Mode 2 (baroclinic or internal mode):

$$\beta_2 = \frac{\epsilon h'}{h+h'}, \quad \tau_2 = i\tau_o \left(\frac{-h'}{h}\right) \quad (7.9.20)$$

$$R_2 = \frac{\sqrt{g\epsilon\frac{hh'}{h+h'}}}{\sqrt{f^2 - \omega^2}} = \frac{\sqrt{g\epsilon\left(\frac{1}{1/h+1/h'}\right)}}{\sqrt{f^2 - \omega^2}} \quad (7.9.21)$$

hence

$$\zeta_2 = \frac{-\frac{h'}{h} \frac{\tau_o}{\rho} \frac{f}{\omega} R_2}{\frac{g\epsilon}{1/h+1/h'}} e^{-x/R_2}$$

Note, $R_2 = O\sqrt{\epsilon}R_1 \ll R_1$.

Clearly

$$\zeta_2 = O\left(\frac{\zeta_1}{\epsilon}\right) \gg \zeta_1 \quad (7.9.22)$$

Recalling from (7.7.51) and (7.7.56) of the last section,

$$\bar{\zeta}_1 \cong \zeta, \quad \bar{\zeta}_2 = -\frac{h'}{h}\zeta + \frac{h+h'}{h}\zeta' \quad (7.9.23)$$

hence the free surface and interface displacements can be solved,

$$\zeta \cong \bar{\zeta}_1 = \Re\left\{\zeta_1 e^{-i\omega t}\right\} = \Re\left\{\frac{\frac{f\tau_0}{\rho}R_1}{\omega g(h+h')}e^{-x/R_1}e^{-i\omega t}\right\} \quad (7.9.24)$$

$$\begin{aligned} \zeta' &\cong \frac{\frac{h'}{h}\bar{\zeta}_1 + \bar{\zeta}_2}{1 + \frac{h'}{h}} = \Re\left\{\frac{1}{1 + \frac{h'}{h}}\frac{h'}{h}\frac{\frac{f\tau_0}{\rho}R_1}{\omega g(h+h')}e^{-x/R_1}e^{-i\omega t}\right\} \\ &+ \Re\left\{\frac{1}{1 + \frac{h'}{h}}\left[\frac{-\frac{h'}{h}\frac{\tau_0}{\rho}\frac{f}{\omega}R_2}{\frac{g\epsilon}{1/h+1/h'}}e^{-x/R_2}\right]e^{-i\omega t}\right\} \end{aligned} \quad (7.9.25)$$

Very close to the coast, $x/R_2 = O(1)$, the internal wave mode dominates.

$$\begin{aligned} \zeta' &\approx \frac{h\bar{\zeta}_2}{h+h'} = \frac{h}{h+h'}\frac{-\frac{h'}{h}\frac{\tau_0}{\rho}\frac{f}{\omega}R_2}{\frac{g\epsilon}{1/h+1/h'}}e^{-x/R_2}\cos\omega t \\ &= -\frac{h'}{h+h'}\frac{\frac{\tau_0}{\rho}\frac{f}{\omega}R_2}{\frac{g\epsilon}{1/h+1/h'}}e^{-x/R_2}\cos\omega t \end{aligned} \quad (7.9.26)$$

Thus as the wind stress is from south to north, $0 < \omega t < \pi$, the interface rises from its lowest (negative) to the highest (positive) level; this is **upwelling**. As the wind reverses direction and blows southward, the interfaces sinks; this is **downwelling**. For $\tau_0 = 0.1Pa$, the upwelling can be several meters.

Farther away from the coast, $x/R_1 = O(1)$, the barotropic (surface wave) mode dominates.

$$\zeta' = \frac{\frac{h'}{h}\bar{\zeta}_1}{\frac{h+h'}{h}} = \frac{h'}{h+h'}\bar{\zeta}_1 = \frac{\frac{f\tau_0}{\rho}R_1}{\omega g(h+h')}e^{-x/R_1}\cos\omega t. \quad (7.9.27)$$

The free surface and the interface rises together when the wind blows northward, and falls together when the wind blows southward. See sketch in Figure 7.9.2.

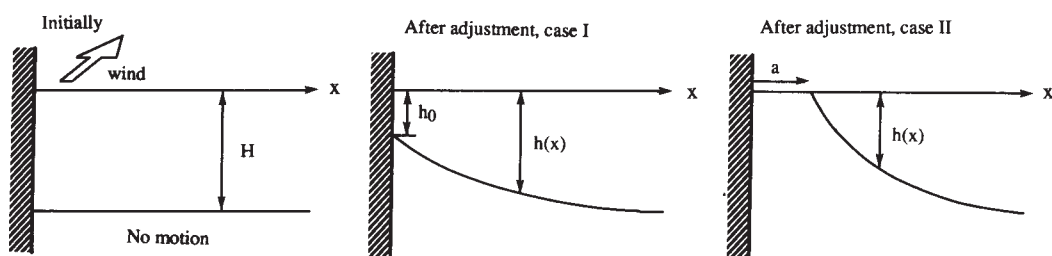


Figure 14-3 The two possible outcomes of coastal upwelling after a longshore wind of finite duration. After a weak or brief wind event (case I), the interface has upwelled but not to the point of reaching the surface. A strong or prolonged wind event (case II) causes the interface to reach the surface, where it forms a front; this front is displaced offshore, leaving cold waters from below exposed to the surface. This latter case corresponds to a mature upwelling that favors biological activity.

Figure 7.9.2: Possible scenarios of coastal upwelling. From Cushman-Roisin.